RISK IN TIME: THE INTERTWINED NATURE OF RISK TAKING AND TIME DISCOUNTING

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Abstract
Standard economic models view risk taking and time discounting as two independent dimensions of decision making. However, mounting experimental evidence demonstrates striking parallels in patterns of risk taking and time discounting behavior and systematic interaction effects, which suggests that there may be common underlying forces driving these interactions. Here, we show that the inherent uncertainty associated with future prospects together with individuals' proneness to probability weighting generates a unifying framework for explaining a large number of puzzling behavioral findings: delay-dependent risk tolerance, aversion to sequential resolution of uncertainty, preferences for the timing of the resolution of uncertainty, the differential discounting of risky and certain outcomes, hyperbolic discounting, subadditive discounting, and the order dependence of prospect valuation. Furthermore, all these phenomena can be accommodated by the same set of preference parameter values and plausible levels of inherent uncertainty. (JEL: D01, D81, D91)

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Teaching Slides
A set of Teaching Slides to accompany this article is available online as Supplementary Data.

1. Introduction

Whatever the nature of our decisions, hardly ever can we be sure about their outcomes. In particular, the consequences of the most important decisions in our lives, such as what line of business to enter or whom to get married to, do not materialize immediately but usually take time to unfold. In other words, these important decisions involve both risk and delay. Driven by the evidence challenging expected utility theory and discounted utility theory, the past half century has seen a surge of new models of decision making for the domains of risk taking and time discounting (Starmer 2000; Frederick, Loewenstein, and O'Donoghue 2002; Wakker 2010; Ericson and Laibson 2019). A considerable body of experimental evidence suggests, however, that risk taking and time discounting are linked and interact with each other in important ways summarized in Table 1.

First, risk aversion has been shown to be lower for risks materializing in the more remote future than for risks materializing in the more imminent future (e.g., Shelley (1994)). Moreover, it seems to be the case that probability weighting rather than utility is the carrier of this effect (Abdellaoui, Diecidue, and Öncüler 2011b). Lower risk aversion for remote risks may be one reason why the mobilization of public support for policies combating global warming is so difficult. Thus, economic models of climate policy may benefit from recognizing that risk aversion decreases with time delay. Asset markets constitute another area where delay-dependent risk aversion may play an important role in understanding the downward sloping structure of risk premia, that is, the fact that risk premia decline with maturity (van Binsbergen, Brandt, and Koijen 2012).

A second regularity is based on a considerable body of evidence that impatience tends to decrease when outcomes are shifted into the more remote future—a finding on which the large literature on hyperbolic discounting is based (e.g., Loewenstein and Thaler (1989)). Hyperbolic discounting has been invoked to explain a large number of phenomena, such as impulsive behavior, procrastination, and insufficient saving for retirement.

Third, the evidence indicates that many people seem to have a preference with respect to the way uncertainty resolves, that is, sequentially or in one-shot. Sequential evaluation of prospects may render decision makers less risk tolerant (e.g., Abdellaoui,

scheme "Welcoming Talents"), and a grant from the I-SITE UNLE (project IBEBACC—funding scheme "Chaire d'Excellence"). The paper builds on the previous working paper Epper and Fehr-Duda (2012) *"The Missing Link: Unifying Risk Taking and Time Discounting"*.

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Klibanoff, and Placido (2015)). In the domain of financial decisions, this phenomenon may underlie the large equity premia observed around the globe.

Fourth, regarding time discounting, a similar phenomenon has been observed: Discount rates compounded over subperiods tend to be higher than the discount rate applied to the total period (e.g., Read (2001)). This incidence of process dependence, labeled *subadditive discounting*, has been put forward as an alternative explanation to hyperbolic discounting to account for the observed patterns in discounting behavior.

Fifth, many people also exhibit a preference regarding the timing of the resolution of uncertainty. Usually, there is a substantial share of participants who favor delayed resolution of uncertainty in situations when money is at stake even though it should be beneficial to know the outcome of one's financial decisions as early as possible (e.g., von Gaudecker, van Soest, and Wengström (2011)). This finding triggered a large theoretical literature following the seminal work of Kreps and Porteus (1978).

A sixth regularity indicates that the presence of risk influences time discounting in an unexpected way: Sure outcomes appear to be discounted more heavily than uncertain ones, discussed in the literature under the heading *diminishing immediacy* (e.g., Keren and Roelofsma (1995)).

Finally, people's evaluations of future risky payoffs depend on the order by which they are devalued for risk and for delay, which should not make any difference according to the standard view (Öncüler and Onay 2009). In particular, discounting for risk first seems to decrease value relative to discounting for time first. All these regularities suggest that theories that are restricted to either domain cannot easily account for the intertwined nature of risk taking and time discounting.

The main purpose of our paper is to provide a unifying account of all these phenomena by integrating risk taking and time discounting into one theoretical approach. Thus, our goal is to present a formal model that is capable of explaining all the regularities on the basis of a parsimonious set of assumptions. Our approach is inspired by a string of papers by Halevy (2008), Saito (2011), Chakraborty, Halevy, and Saito (2020), and rests on two key assumptions: First, there is risk attached to any future prospect because only immediate consequences can be totally certain. We believe that this is a plausible assumption because it is impossible to foresee all future contingencies. Accordingly, Prelec and Loewenstein (1991) state that "anything that is delayed is almost by definition uncertain" (p. 784). In particular, it is always possible that an event may occur that prevents the realization of a future outcome, that is, something may go wrong before payoffs actually materialize. An unforeseen event may arise, such as missing one's transatlantic flight because the taxi driver was late, or realizing that one has forgotten one's passport at home. Presumably, almost everyone can readily recall such an incident. We capture the notion that something may go wrong by introducing a survival probability $0 < s < 1$ that applies also to allegedly certain future outcomes.

Second, if future prospects are perceived as inherently risky, people's risk tolerance must play a role in their valuations of future prospects. Therefore, the characteristics of (atemporal) risk preferences are crucial not only for evaluating delayed risky prospects but also for delayed (allegedly) certain ones. There is abundant evidence from the

TABLE 1. Observations on risk taking and time discounting.

Depends on		Risk tolerance		Patience
Delay	#1	Increases with delay	#2	Increases with delay
Process	#3	Higher for one-shot than sequential valuation	#4	Higher for one-shot than sequential valuation
Timing	#5	Higher for late than immediate resolution		–
Risk		–	#6	Higher for risky payoffs than for certain ones
Order	#7	Higher for time-first than risk-first order		–

Notes: The table describes seven regularities in experimental findings on risk taking and time discounting behavior with respect to delay, process, timing, risk, and order effects. In Section 5, we present a comprehensive discussion of references regarding the empirical evidence and extant theories that address various subsets of these findings. An overview is provided in Online Appendix A.

field and the laboratory that risk taking behavior depends nonlinearly on the objective probabilities (Prelec 1998; Fehr-Duda and Epper 2012; Barberis 2013; O'Donoghue and Somerville 2018). For this reason, models involving probability weighting, such as rank-dependent utility theory (RDU) (Quiggin 1982) and cumulative prospect theory (Tversky and Kahneman 1992) have been strong contenders of expected utility theory (EUT) (Wakker 2010).

Our approach relies on a key characteristic of probability weighting, proneness to Allais-type *common ratio violations*, that is, one of the most widely replicated experimental regularities, found in human and animal behavior: Probabilistically mixing two lotteries with an inferior lottery frequently leads to preference reversals (Kahneman and Tversky 1979; Gonzalez and Wu 1999). This feature of probability weighting is called *subproportionality* and was characterized axiomatically by Prelec (1998).

Our contribution to the literature is threefold. First, we show for general m-outcome prospects that subproportional probability weighting under RDU together with the assumption that (even allegedly certain) future outcomes are inherently risky provides an integrative account of *all* the above mentioned experimental regularities. We rely on a well-established model of risk preferences with axiomatic foundations (e.g., Quiggin (1982)), that we combine with the plausible assumption that something may go wrong in the future.

In particular, our theoretical contribution builds on and explores the ramifications of Halevy (2008)'s ideas, later clarified and extended by Saito (2011) and Chakraborty, Halevy, and Saito (2020).[1] Our objective is to demonstrate that the consequences of their assumptions are not limited to the delay and risk dependence of patience, Observations #2 and #6 in Table 1, but provide the basis for unifying all the seven experimental regularities listed there. We derive novel predictions regarding

1. Relatedly, Chakraborty (2021) explores risk-time separability violations by adopting a weaker version of the stationarity axiom to simple risky prospects $(x, p; 0, 1-p)$.

(i) the delay dependence of probability weights and (ii) the intrinsic preference for late resolution of uncertainty. Furthermore, we take advantage of Segal's (Segal 1987a, b, 1990) and Dillenberger (2010)'s contributions to sequential prospect evaluation by explicitly integrating the dimension of time delay and show that (iii) subproportional risk preferences imply subadditive discounting and (vi) that, under certain circumstances, an aversion to sequential resolution of uncertainty arises and remains intact under inherent future uncertainty.

Second, we demonstrate that the same set of preference parameter values together with a narrow and plausible range of survival probabilities provide a reasonable *quantitative* account of all seven phenomena. Furthermore, we provide novel evidence that, at the individual level, reported perceptions of future uncertainty are indeed significantly related to the estimated values of the survival probabilities. Thus, this evidence not only underscores the credibility of our assumptions but also substantiates that the perception of uncertainty is actually an important mechanism underlying observed behavior.

Third, we derive new predictions, which can be put to the test by future research. We show, for example, that the decrease in risk tolerance, induced by the sequential resolution of uncertainty of (atemporal) prospect risks, carries over, under certain conditions, to the sequential resolution of uncertainty for delayed future prospects. This prediction is important as many societal risks (e.g., climate risks) and asset market risks resolve sequentially over time. However, so far this prediction has not been tested experimentally.

The remainder of the paper is organized as follows: Section 2 discusses the key assumptions of our model. Its implications for explaining the seven types of findings are developed in Section 3. Section 4 is devoted to a quantitative assessment of our model predictions and the exploration of the relationship between reported and estimated levels of future uncertainty. Section 5 presents the experimental findings on the seven phenomena and discusses other theoretical approaches that address some of these empirical regularities. Finally, Section 6 concludes. Propositions including proofs and complementary materials are available in the Online Appendix.

2. The Model

In the following, we will first present the general setup of our approach. Second, we justify our assumptions on the characteristics of the probability weighting function. Finally, we explain how we integrate that "something may go wrong" into the model.

2.1. Risk Preferences

In this paper, we rely on RDU, a generalization of EUT, that allows for nonlinear weighting of the probabilities, which has proven to be an exceptionally powerful component for capturing deviations from EUT (Diecidue and Wakker 2001). According to RDU, a decision maker's atemporal risk preferences over prospects that are played out and paid out with negligible time delay can be represented by a rank-dependent functional. Consider a prospect $P = (x_1, p_1; \ldots; x_m, p_m)$ over (terminal) monetary outcomes $x_1 > x_2 > \ldots > x_m$ with $x_i \in X \subset \mathbb{R}$, $p_i \in [0, 1]$ and $\Sigma p_i = 1$. The function u denotes the utility of monetary amounts x, and w denotes the subjective probability weight attached to p_1, the probability of the best outcome x_1. As usual, both u and w are assumed to be monotonically increasing, w to be twice differentiable on (0,1) and to satisfy $w(0) = 0$ and $w(1) = 1$. Decision weights π_i are defined as[2]

$$\pi_i = \begin{cases} w(p_1) & \text{for } i = 1 \\ w\left(\sum_{k=1}^{i} p_k\right) - w\left(\sum_{k=1}^{i-1} p_k\right) & \text{for } 1 < i \le m \end{cases}. \tag{1}$$

Thus, the decision weight of x_i is the probability weight attached to the probability of obtaining something at least as good as x_i subtracted by the probability weight attached to the probability of obtaining something strictly better than x_i. Consequently, decision weights sum to 1. Finally, the prospect's value is represented by

$$V(P) = \sum_{i=1}^{m} u(x_i)\pi_i. \tag{2}$$

To keep the logic of our approach as transparent as possible, we present the following steps for $m = 2$ and delegate the general case to Online Appendix B.1. For $m = 2$, the prospect reduces to $P = (x_1, p; x_2, 1 - p)$ and equation (2) reads as

$$\begin{aligned} V(P) &= u(x_1)w(p) + u(x_2)(1 - w(p)) \\ &= (u(x_1) - u(x_2))w(p) + u(x_2). \end{aligned} \tag{3}$$

This representation of V clarifies that x_2 is effectively a sure thing, whereas obtaining something better than x_2 is risky.

If the prospect is not played out and paid out in the present, but at some future time $t > 0$, prospect value is affected by time discounting as well. We follow the standard approach and model people's willingness to postpone gratification by a constant rate of time preference $\eta \geq 0$, yielding a discount weight of $\rho(t) = \exp(-\eta t)$.[3] A prospect to be played out and paid out at $t > 0$ is discounted for time in the following standard way:

$$V_0(P) = V(P)\rho(t). \tag{4}$$

Abundant empirical evidence has demonstrated that risk taking behavior depends nonlinearly on the probabilities (Starmer 2000; Fehr-Duda and Epper 2012). However, in order to explain the observed interaction effects, we need to put more structure on the type of nonlinearity.

2. Alternatively, decision weights π_i can be expressed in terms of the cumulative distribution function F of the outcomes x_i: $\pi_i = w(1 - F(x_{i+1})) - w(1 - F(x_i))$ for $1 \le i \le m$, where $F(x_{m+1}) := 0$.

3. This assumption is not crucial for our results—neither a 0 rate of time preference, that is, $\rho = 1$, nor genuinely hyperbolic time preferences affect our conclusions.

2.2. *Probability Weighting*

Our approach is based on proneness to *common ratio violations*, originally brought to the fore by Allais (1953).[4] In RDU, common ratio violations are mapped by subproportionality of probability weights. Formally, *subproportionality* holds if $1 \geq p > q > 0$ and $0 < \lambda < 1$ imply the inequality

$$\frac{w(p)}{w(q)} > \frac{w(\lambda p)}{w(\lambda q)}, \tag{5}$$

(Prelec 1998).[5]

Subproportionality implies the *certainty effect*, which constitutes the special case of $p = 1$. Therefore,

$$w(\lambda q) > w(\lambda)w(q), \tag{6}$$

is satisfied for any λ, q such that $0 < \lambda, q < 1$. This feature of subproportional probability weighting has a crucial consequence: It produces an aversion to compounding of probability weights (Segal 1987a, b, 1990). We will use this insight when we discuss aspects of uncertainty resolution.

When inspecting the graph of $w(p)$, one cannot detect subproportionality with the naked eye. In fact, many different shapes of $w(p)$ display subproportionality, at least over some range of probabilities. Figure 1 depicts three examples of globally subproportional probability weighting functions with starkly different shapes: an inverse S-shaped, a concave, and a convex function.

Aside from the examples in Figure 1, many other functional specifications have been proposed in the literature (see Online Appendix E.3). Perhaps the most widely used representative of a globally subproportional function is Prelec (1998)'s flexible two-parameter specification of the compound invariant class, designed to map common ratio violations. This functional form is particularly useful because it provides a direct measure of subproportionality. Therefore, we will use this "standard" functional specification throughout the paper to illustrate our results,[6] defined as

$$w(p) = \exp\big(-\beta(-\ln(p))^{\alpha}\big), \tag{7}$$

where $0 < \alpha$ governs the departure from linearity and $0 < \beta$ governs the range of convexity. The function is subproportional (supraproportional) for $\alpha < 1$ ($\alpha > 1$) with

4. For example, many people prefer $(30, 1)$ to $(40, 0.8; 0, 0.2)$, but prefer $(40, 0.2; 0, 0.8)$ to $(30, 0.25; 0, 0.75)$, that is, scaling down the probabilities by a common factor leads to preference reversals that are inconsistent with EUT. An intuitive explanation for common ratio violations is based on emotional reactions (Wu 1999; Walther 2003).

5. Kahneman and Tversky (1979) note that this property imposes considerable constraints on the shape of w: It holds if and only if $\ln w$ *is a convex function of* $\ln p$. In other words, $((d \ln w)/(d \ln p))' > 0$, or the elasticity of w, $\varepsilon_w(p) = (d \ln w)/(d \ln p)$, is increasing in p. The equivalence of subproportionality and increasing elasticity is shown in Online Appendix E.1.

6. Aydogan, Bleichrodt, and Gao (2016) provide experimental support for the compound invariant specification at the level of preference conditions.

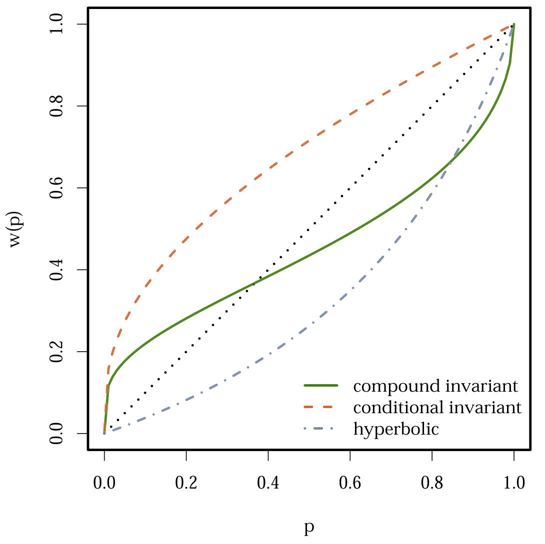

FIGURE 1. Exhibits of subproportional probability weighting functions. **Compound invariant:** $w(p) = \exp(-(-\ln(p))^{0.5})$ (Prelec 1998). This function is globally subproportional and inverse S-shaped. **Conditional invariant:** $w(p) = \exp(-5(1 - p^{0.1}))$ (Prelec 1998). This function is globally subproportional and concave. **Hyperbolic:** $w(p) = p/p + 2.8(1 - p)$ (Rachlin, Raineri, and Cross 1991). This function is globally subproportional and convex.

smaller (larger) values of α indicating more pronounced degrees of subproportionality (supraproportionality). The greater is β, the greater, ceteris paribus, is the range of probabilities for which the curve is convex, that is, underweighting p.

2.3. Future Uncertainty

The final building block of our model concerns the integration of "something may go wrong" due to events unrelated to the prospect under consideration. This (uninsurable) risk inherent in the future, *survival risk* for short, turns allegedly guaranteed payoffs into risky ones, and introduces an additional layer of risk over and above the objective probability distributions of risky payoffs (henceforth, referred to as *prospect risk*). Consequently, there are two distinct types of risk, *prospect risk*, which may resolve at any time between the present and the payment date, and *survival risk*, which resolves fully only at the payment date. Thus, the subjective perception of future uncertainty changes the nature of the prospect. Formally, let $s < 1$ denote the constant per-period probability of prospect survival, that is, the probability that the decision maker will actually obtain the promised rewards by the end of the period. Essentially, there are two ways of accounting for this subjective probability s. First, for a delay t, the probability s^t is transformed according to the decision maker's probability weighting function, and the resulting $w(s^t)$ affects the prospect as a whole, that is, all outcomes equally.

In this case, prospect value amounts to

$$V_0(P) = V(P)w(s^t)\rho(t). \tag{8}$$

Such an approach only affects measured discount rates but cannot handle the observed interaction effects. Thus, we work with the second solution, namely, that s impacts the perceived probability distribution of the prospect, as originally analyzed by Halevy (2008). In that case, the probability that the allegedly guaranteed payment x_m materializes at the end of period t is perceived to be s^t, and the probabilities of obtaining something better than x_m are scaled down by s^t. Therefore, the objective m-outcome prospect is subjectively perceived as an (m+*1*)-outcome prospect. Focusing on $m = 2$ again, $\widetilde{P} = (P, s^t; \underline{x}, 1 - s^t) = (x_1, ps^t; x_2, (1-p)s^t; \underline{x}, 1 - s^t)$, where $\underline{x} < x_m$ captures that "something may go wrong."

Setting $u(\underline{x}) = 0$, the subjective present value of the prospect amounts to

$$\begin{aligned} V_0(\widetilde{P}) &= \big((u(x_1) - u(x_2))w(ps^t) + u(x_2)w(s^t)\big)\rho(t) \\ &= \big((u(x_1) - u(x_2))\tfrac{w(ps^t)}{w(s^t)} + u(x_2)\big)w(s^t)\rho(t). \end{aligned} \tag{9}$$

From the point of view of an outsider, the subjective probability distribution of prospect $\tilde{P}$ is not observable. Consequently, she infers probability weights $\widetilde{w}$ and discount weights $\tilde{\rho}$ from observed behavior on the presumption that the decision maker evaluates the objectively given prospect P, and estimates preference parameters according to RDU in the standard way[7]:

$$V_0(\widetilde{P}) = \big((u(x_1) - u(x_2))\widetilde{w}(p) + u(x_2)\big)\tilde{\rho}(t), \tag{10}$$

interpreting observed $\widetilde{w}$ as true probability weights and observed $\tilde{\rho}$ as true discount weights, while in fact the observed weights are distorted by survival risk. By comparing equation (9) with equation (10), we can see that the relationships between true underlying weights and observed ones are given by

$$\widetilde{w}(p) = \frac{w(ps^t)}{w(s^t)}, \tag{11}$$

and

$$\tilde{\rho}(t) = w(s^t)\rho(t)\,. \tag{12}$$

Since $\widetilde{w}(p) \neq w(p)$ and $\tilde{\rho}(t) \neq \rho(t)$ under subporportionality, the presence of survival risk drives a wedge between true underlying preferences and observed risk taking and discounting behavior. Thus, future risk conjointly with proneness to Allais-type behavior provides the mechanism by which behavior under risk and behavior over time are intertwined. A summary of the model variables is provided in Table 2.

7. Note that it takes at least two non-zero outcomes to separate risk taking and time discounting.

TABLE 2. Model variables.

	Variable	Description	Characteristics
Prospects	x	Monetary payoff	$x \geq 0$
	p	Probability of x	$0 \leq p \leq 1$
	s	Probability of prospect survival	$0 \leq s < 1$
	$1 - s$	Survival risk	”
	t	Length of time delay	$t \geq 0$
Preferences	$u(x)$	Utility function	$u(0) = 0, u' > 0$
	$w(p)$	True probability weight	$w(0) = 0, w(1) = 1, w' > 0$
	η	Rate of pure time preference	$\eta \geq 0$, constant
	$\rho(t)$	Discount weight	$\rho(t) = \exp(-\eta t)$
Behavior	$\widetilde{w}(p)$	Observed probability weight	$\widetilde{w}(p) = \frac{w(ps^t)}{w(s^t)}$
	$\tilde{\rho}(t)$	Observed discount weight	$\tilde{\rho}(t) = w(s^t)\rho(t)$
	$\tilde{\eta}(t)$	Observed discount rate	$\tilde{\eta}(t) = -\frac{\tilde{\rho}'(t)}{\tilde{\rho}(t)}$

3. Model Predictions: Unifying the Experimental Evidence

In the following, we discuss the implications of our approach for the experimental phenomena listed in Table 1 and demonstrate that, qualitatively, all the Observations #1 through #7 can be explained within our framework. A quantitative assessment of the model's performance is presented in Section 4. While we retain some of the fundamental calculations in the main text, propositions, and their proofs are presented in Online Appendix B.

3.1. Prediction #1: Delay Dependence of Risk Tolerance

The first fact in our list considers the observation that risk tolerance for delayed prospects seems to be higher than risk tolerance for present ones. Concerning delayed risky prospects, we examine the case when prospect risk and survival risk are resolved simultaneously in one-shot at time t. We have seen from equation (11) that observed probability weights $\widetilde{w}(p)$ deviate systematically from the underlying atemporal ones $w(p)$,

$$\widetilde{w}(p) = \frac{w(ps^t)}{w(s^t)}\,.$$

As $w(s^t) < 1$, the denominator boosts observed probability weights, whereas the additional s^t in the argument of w in the numerator distorts them. Due to subproportionality

$$\widetilde{w}(p) = \frac{w(ps^t)}{w(s^t)} > \frac{w(p)}{w(1)} = w(p), \tag{13}$$

implying that $\widetilde{w}$ is more elevated than w, that is, that $\widetilde{w}$ lies above w, which constitutes one of the central implications of our model. Moreover, the wedge between $\widetilde{w}$ and w

increases with t. Since the probability weighting function maps the decision weight of the best possible outcome, an increase in the elevation of the probability weighting curve gets directly translated into higher revealed risk tolerance. For $m = 2$ and a given observed discount weight $\tilde{\rho}(t) = w(s^t)\rho(t)$,

$$\begin{aligned} V_0(\widetilde{P}) &= \big((u(x_1) - u(x_2))\widetilde{w}(p) + u(x_2)\big)w(s^t)\rho(t) > \\ V_0(P) &= \big((u(x_1) - u(x_2))w(p) + u(x_2)\big)w(s^t)\rho(t). \end{aligned} \tag{14}$$

Thus, the presence of survival risk makes people appear more risk tolerant for delayed prospects than for present ones. Intuitively, the event of something going wrong takes on the role of the perceived sure outcome, which makes x_2 an intermediate one and, thus, less salient to the decision maker. In addition, this risk-tolerance increasing effect is particularly strong for small probabilities, that is, positively skewed prospects are subject to more pronounced increases in risk tolerance, as $\widetilde{w}(p)/w(p)$ declines in p (see Proposition 1 in Online Appendix B.2). Such a prediction would not be possible if the utility function were the carrier of delay dependence, as for instance in Eisenbach and Schmalz (2016).

Our prediction is also at odds with Baucells and Heukamp (2012)'s model, which features a time-dependent probability weighting function $g(p,t) = g(p\exp(-r_x t))$, where r_x denotes a probability discount rate. This probability discount rate looks prima vista similar to the probability of prospect survial s in our model, but unlike s, also depends on outcome magnitude x. Furthermore, in our model the probability weighting function

$$\widetilde{w}(p) = \frac{w(p\exp(-(-\ln s)t))}{w(\exp(-(-\ln s)t))},$$

additionally involves the denominator $w(\exp(-(-\ln(s))t))$, which ensures that $\widetilde{w}$ increases in t, whereas g decreases in t. Thus, Baucells and Heukamp (2012) can only explain that risk tolerance increases in t if they invoke an additional assumption, namely, that r_x decreases in x.

The delay dependence of observed probability weights $\widetilde{w}$ is illustrated in Figure 2. The top row of the figure characterizes preferences in the atemporal case. Panel 1(a) shows a typical specimen of a subproportional probability weighting function w for $t = 0$, underweighting large probabilities and overweighting small probabilities of the best outcome. For illustrative purposes, Panel 1(b) on the right side depicts the corresponding decision weights π_i for a prospect involving 21 equiprobable outcome levels, with outcome rank 1 denoting the best outcome and outcome rank 21 denoting the worst one. Their objective probabilities $p_i = 1/21$ are represented on the horizontal gray line. As one can see, w generates strong overweighting of the extreme outcomes and underweighting of the intermediate ones relative to the objective probability distribution.

The bottom row of Figure 2 demonstrates the predictions for the case when prospects are played out and paid out simultaneously in the future, the focus of this section. Future uncertainty is captured by the parameter $s = 0.8$, that is, the per-period prospect survival rate is perceived to be 80%. When payoffs are delayed by

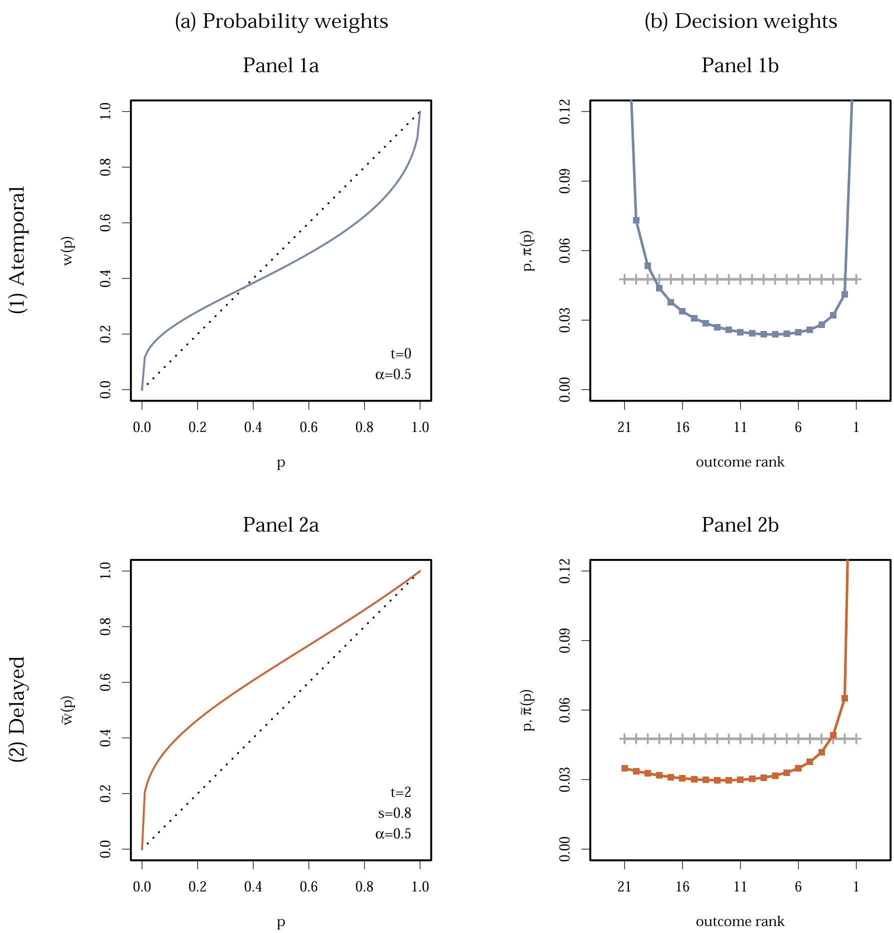

FIGURE 2. Prediction #1. Delay dependence of risk tolerance. The figure contrasts atemporal probability and decision weights with weights delayed by $t = 2$ periods. For purposes of illustration, the probability weighting curves are derived from Prelec (1998)'s two-parameter probability weighting function $w(p) = \exp(-\beta(-\ln(p))^{\alpha})$, assuming degrees of subproportionality $\alpha = 0.5$ and of convexity $\beta = 1$. Survival risk s is set at 0.8 per period. **Top row—(1) Atemporal:** The graphs show atemporal probability weights w (Panel 1(a)) and their associated decision weights π (Panel 1(b)) for a prospect involving 21 equiprobable outcomes, with outcome rank 1 denoting the best outcome. Their objective probabilities are represented on the horizontal gray line. **Bottom row—(2) Delayed:** Panel 2(a) and 2(b) show $\widetilde{w}$ and $\tilde{\pi}$ for a delay of two periods when uncertainty resolves at $t = 2$.

two periods, $t = 2$, observed probability weights $\widetilde{w}$ shift upwards, as shown in Panel 2(a). This shift rotates the decision weights $\tilde{\pi}_i$ counterclockwise, as depicted in Panel 2(b). Now the worst outcomes are underweighted, while the best ones are more strongly overweighted. For longer time delays, these effects become more pronounced and may lead to a substantial underweighting of the worst outcomes. Thus, underweighting of adverse extreme events becomes more pronounced with longer time horizons.

3.2. *Prediction #2: Delay Dependence of Patience*

The following section picks up the main topic of Halevy (2008)'s work and is dedicated to the fact that observed discount rates decrease with the length of delay, that is, exhibit a hyperbolic decline. Allegedly guaranteed future payoffs constitute a special case of risky ones. As is evident from equation (12), the observed discount weight for time equals $\tilde{\rho}(t) = w(s^t)\rho(t)$. Clearly, if w is linear, $\tilde{\rho}$ declines exponentially irrespective of the magnitude of s. To see this, note that $\rho(t) = \exp(-\eta t)$ and $s^t = \exp(-(-\ln(s))t)$, implying a discount rate $\tilde{\eta} = \eta - \ln(s) > \eta$ for $s < 1$. In this case, uncertainty *per se* increases the absolute level of revealed impatience, but it cannot account for declining discount rates. Thus, an expected utility maximizer will exhibit a constant discount rate that is higher than her underlying rate of pure time preference, but her behavior will not show any of the interaction effects addressed in this paper. If, however, w is subproportional and $s < 1$, the component $w(s^t)$ distorts the discount weight in a predictable way (see details in Proposition 2 in Online Appendix B.3): The discount function $\tilde{\rho}(t)$ declines at a decreasing rate, that is, in a hyperbolic way. To show this result, we set $\rho = 1$ without loss of generality. The rate $\tilde{\eta}(t)$ at which $w(s^t)$ declines is defined as

$$\tilde{\eta}(t) = -\frac{\frac{\partial w(s^t)}{\partial t}}{w(s^t)} = -\frac{w'(s^t)s^t \ln s}{w(s^t)} = -\varepsilon(s^t)\ln s, \tag{15}$$

where ε denotes the elasticity of w. Note that subproportionality of w is equivalent to increasing elasticity. Therefore,

$$\tilde{\eta}'(t) = -\varepsilon'(s^t)s^t(\ln s)^2 < 0, \tag{16}$$

since the elasticity of w is increasing. As Chakraborty, Halevy, and Saito (2020) have clarified, subproportionality not only predicts hyperbolic discounting, but the reverse relationship also holds in our setting.

Thus, decreasing impatience is not necessarily a manifestation of pure time preferences but a consequence of survival risk changing the subjective nature of future prospects. At the level of observed behavior, decreasing impatience is the mirror image of increasing risk tolerance if survival risk is integrated into the prospect's probability distribution. In fact, the degree of proneness to common ratio violations, the degree of subproportionality, can be interpreted as the degree of time insensitivity. Because more immediate payoffs are more likely to actually materialize than more remote payoffs, this potential is perceived to decline with the passage of time and becomes almost negligible for payoffs far out in the future. Technically, since shifting a payoff into the future amounts to scaling down its probability, which constitutes an intertemporal variant of the Allais common ratio effect, a decision maker with subproportional preferences becomes progressively insensitive to a given timing difference. This insight provides a test bed for analyzing risk taking and time discounting behavior at the individual level because the characteristics of the probability weighting function feed directly into the characteristics of the observed discount function. For example, a Prelec compound invariant probability weighting function with $\alpha < 1$ generates a

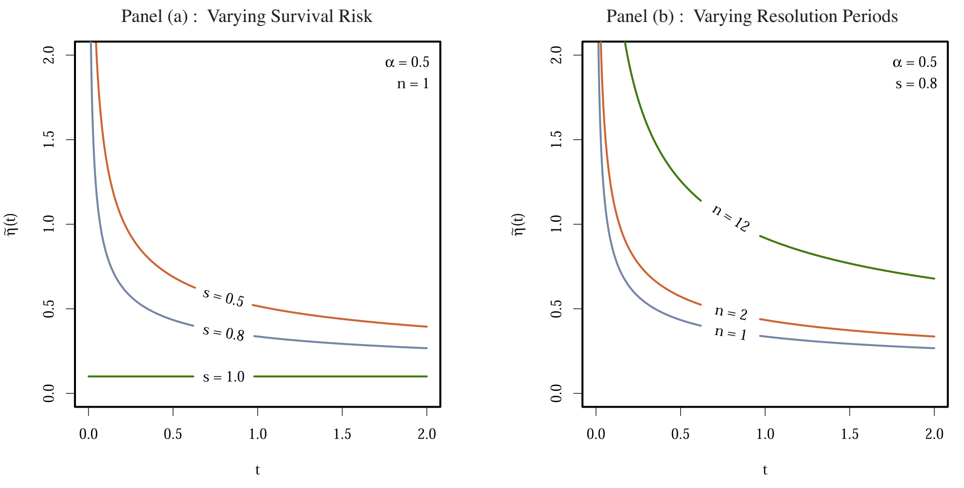

FIGURE 3. Predictions #2 and #4. Hyperbolic and subadditive discount rates $\tilde{\eta}$. Panel (a) shows discount rates as they move with the length of delay t for different levels of survival risk $1-s$, where s denotes the probability of prospect survival. When there is no survival risk, $s=1$, the observed discount rate is constant and equals the rate of pure time preference (line labeled by $s=1.0$). The higher is the level of risk, the lower s, the more pronounced the hyperbolic decline of discount rates over time is for decision makers with subproportional probability weights (curves labeled by $s=0.5$ and $s=0.8$). $\tilde{\eta}(t) := -\frac{\partial\tilde{\rho}}{\partial t}/\tilde{\rho}$. w is specified as Prelec's probability weighting function (in this example $\alpha=0.5$ and $\beta=1$). Panel (b) depicts discount rates for a constant level of survival probability $s=0.8$ and varying number of resolution stages n. The more often a particular delay is divided into subintervals (of equal length in this graph), the higher is the discount rate, a manifestation of subadditive discounting.

Constant Relative Decreasing Impatience (CRDI) discount function, frequently used to map hyperbolic discounting (Bleichrodt, Rohde, and Wakker 2009).

The effects of survival risk on revealed discount rates are presented in Panel (a) of Figure 3, which depicts a typical decision maker's observed *discount rates* $\tilde{\eta}$ as they react to varying levels of s. The horizontal line represents the case of no survival risk, $s=1$. In this case, the observed discount rate $\tilde{\eta}$ is constant and coincides with the true underlying rate of time preference η. When survival risk comes into play, however, discount rates decline in a hyperbolic fashion, and depart from constant discounting increasingly strongly with rising uncertainty, as shown by the curves for $s=0.8$ and $s=0.5$, respectively.

3.3. *Prediction #3: Process Dependence of Risk Tolerance*

So far, we have considered the case when future prospects are evaluated in one single shot. In the following section, we analyze the situation of uncertainty resolving in several distinct stages. In the domain of risk, sequential resolution of uncertainty frequently reduces a prospect's value relative to its one-shot counterpart, Observation

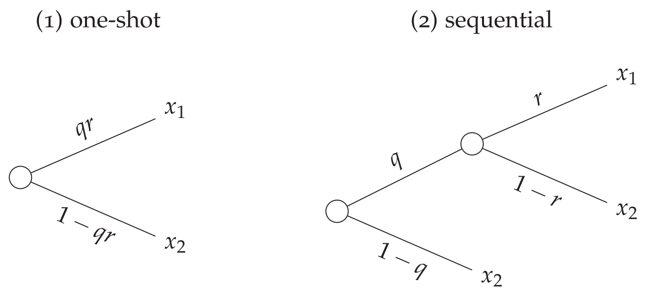

FIGURE 4. One-shot and sequential resolution of prospect risk. (1) One-shot: The probability tree depicts uncertainty resolution in one stage. (2) Sequential: The probability tree shows the sequential resolution of uncertainty of a prospect $P = (x_1, qr; x_2, 1 - qr)$ in two stages with partial probabilities q and r.

#3. We will first analyze the atemporal case and introduce the passage of time in the ensuing subsection.

3.3.1. Process Dependence in the Atemporal Case. In order to derive our predictions for sequential resolution of uncertainty, we need to discuss the method by which multi-stage prospects are transformed into single-stage ones, the domain over which risk preferences are commonly defined. In principle, there are two different transformation methods, reduction by probability calculus and folding back.[8] Reduction involves the calculation of the probabilities of the final outcomes and the transformation of these values by the appropriate weighting function. Folding back, on the other hand, weights the probabilities at each stage and then compounds these weights. Segal (1990) argues that folding back is particularly plausible when the stages are clearly distinct. It is well known that a naive RDU decision maker will be dynamically inconsistent if she cares only about the probabilities of the final outcomes—as the payment date draws near, she will re-evaluate the prospect and, because of the delay dependence of risk tolerance, become comparatively more risk averse. Folding back ensures dynamic consistency but has substantial consequences for prospect valuation.

Experiments on compound risks show that people frequently violate the reduction axiom of EUT, that is, the value of a prospect resolving in several stages differs from the value of the probabilistically equivalent one-stage prospect.[9] In the following, we assume that the decision maker applies folding back when evaluating the prospect under consideration.

Figure 4 depicts the sequential resolution of a two-outcome prospect $P = (x_1, p; x_2, 1 - p)$ in $n = 2$ stages with partial probabilities q and r and the corresponding one-shot resolution case. Under folding back, the prospects' values are

8. Segal (1990) replaces the reduction axiom by an axiom of compound independence, which ensures the applicability of folding back as a transformation mechanism.

9. This violation of the reduction axiom is not necessarily a manifestation of bounded rationality, but may be an expression of a genuine preference (Wakker 1988; Segal 1990).

given by

$$V_1(P) = (u(x_1) - u(x_2))w(qr) + u(x_2),$$

and

$$V_2(P) = (u(x_1) - u(x_2))w(r)w(q) + u(x_2),$$

where the subscripts of V denote the number of resolution stages. As already noted, and discussed in detail in Segal's work (Segal 1987a, b, 1990), the certainty effect embodied in subproportional preferences generates an aversion to compounded probability weights: For $1 > p = qr > 0$, the compounding of the respective weights always leads to lower prospect values, that is, $w(qr) > w(q)w(r)$ holds whatever are the values of q and r. Here, the order of r and q, that is, which probability resolves first, does not play a role, a feature labeled *event commutativity* (Chung, von Winterfeldt, and Luce 1994). Furthermore, a prospect's minimum value is attained when compounding occurs over equiprobable stages, that is, when $r = q = \sqrt{p}$. Partitions of equal length correspond to the least degenerate multi-stage prospect and can be interpreted as the comparatively most vague situation, which is strongly disliked by people with subproportional preferences.[10]

When the prospect under consideration is more complex than the 2-outcomes-2-stages case, uncertainty may resolve in many different ways. This raises the question whether subproportionality always implies a preference for one-shot resolution of uncertainty, irrespective of the resolution pattern. As Dillenberger (2010) has shown, a preference for one-shot resolution of uncertainty does not hold generally in RDU with subproportionality. For details see our discussion in Online Appendix E.2. Here, we focus on sequential resolution in the form of prospect survival, that is, we only consider probability trees that, at each stage, render either x_2 or the chance that x_1 is still available at a later stage. We term trees with this structure "survival trees". For example, a third stage with partial probability v could be appended to the tree in Figure 4, such that x_1 materializes with probability qrv. For survival trees with $m = 2$ outcomes, Segal's insights on two-stage prospects generalize to $n > 2$ stages, that is, $w(\prod_{i=1}^{n} q_i) > \prod_{i=1}^{n} w(q_i)$ for $\prod_{i=1}^{n} q_i = p$, as shown in Proposition 3 in Online Appendix B.5.

For $m > 2$, another type of survival tree emerges when, at each stage, either the worst possible outcome materializes or "everything is still possible," which could be any number of probabilistic outcomes that materialize at the final stage. Thus, the survival tree has two branches at all the chance nodes before the final stage, and m branches at the terminal resolution of uncertainty. An example for $m = 3$ outcomes and $n = 3$ stages is discussed in Online Appendix B.4. Subproportionality makes clear

10. Because of this characteristic, Segal (1987b) proposes to model ambiguity aversion by subproportional risk preferences over two-stage lotteries. Dillenberger and Segal (2014) show that such an approach has another attractive implication: It is able to solve Machina (2009, 2014)'s paradoxes, which involve a number of situations where standard models of ambiguity aversion are unable to capture plausible features of ambiguity attitudes (Baillon, l'Haridon, and Placido 2011).

predictions for this type of sequential resolution of uncertainty as well: The prospect's one-shot value will be greater than its folded back version. Thus, such a resolution process has the flavor of disappointment aversion since at each stage something better than x_m may turn out to be unreachable. For n resolution stages and m outcomes, the resulting probability weighting function for $\prod_{i=1}^{n} q_i = p$ is given by

$$w_n(p) = \prod_{i=1}^{n} w(q_i). \tag{17}$$

Details are set out in Proposition 3 in Online Appendix B.5.

The top row of Figure 5 shows the basic probability weighting function and the decision weights of 21 equiprobable outcomes when uncertainty resolves in one-shot. On the bottom, the probability weighting function and the corresponding decision weights are displayed that result from compounding over twelve stages of equal partial probability when uncertainty resolves along a survival tree. As one can see, the originally inverse S-shaped probability weighting function is transformed into a strongly convex one. The decision weight curve now rotates clockwise, implying substantial underweighting of the best outcomes and overweighting of the worst outcomes, as is evident in Panel 2(b). Thus, compounding probability weights greatly reduces risk tolerance. Sequential valuation of this type, therefore, has a dramatic effect on the overweighting of adverse tail events. This effect may be called *myopic probability weighting* in the style of myopic loss aversion (Benartzi and Thaler 1995), which has similar consequences on risk taking behavior when short-sighted investors are frequently exposed to the possibility of facing losses.

To sum up: If uncertainty resolves according to a survival tree, under subproportionality, one-shot resolution is always preferred to sequential resolution of uncertainty.

3.3.2. Process Dependence of Risk Tolerance and the Passage of Time. The property of aversion to compound risk carries over to the case when the passage of time with its inherent uncertainty is introduced. In our view, this situation constitutes a much more interesting case than the frequently observed aversion to sequential resolution in atemporal experimental settings. However, we are not aware of any studies involving the sequential resolution of uncertainty of genuinely delayed prospects. Thus, the following insights provide the basis for novel experimental investigations.

In the following, we set $\rho = 1$ for ease of exposition. Let us first consider a two-outcome prospect $P = (x_1, p; x_2, 1 - p)$ resolving in $n = 2$ stages denoted by corresponding subscripts to $\tilde{w}$ and $\tilde{\rho}$, such that uncertainty is partially resolved at some future time t_1 and fully resolved at the payment date $t > t_1$, as depicted in Panel (ii) of Figure 6.[11] Applying folding back, the resulting two-stage prospect is evaluated as

11. A more complex survival tree is displayed in Figure B.2 in Online Appendix B.4.

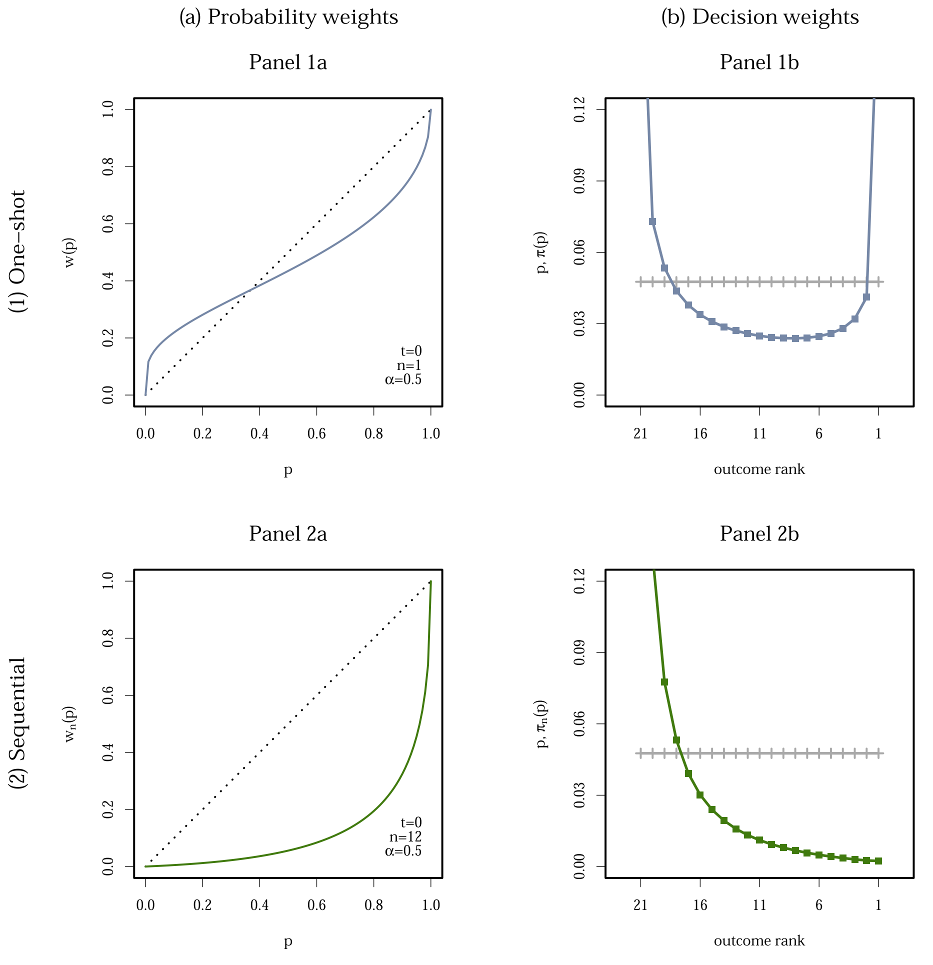

FIGURE 5. Prediction #3. Preferences for one-shot resolution of atemporal uncertainty. The figure contrasts probability and decision weights for one-shot resolution of uncertainty with the weights for sequential resolution along a survival tree if the passage of time does not play a role. For purposes of illustration, the curves are derived from Prelec (1998)'s two-parameter probability weighting function $w(p) = \exp(-\beta(-\ln(p))^{\alpha})$, assuming degrees of subproportionality $\alpha = 0.5$ and of convexity $\beta = 1$. Top row—(1) One-shot: The graphs show probability weights w (Panel 1(a)) and their associated decision weights π (Panel 1(b)) for a prospect involving 21 equiprobable outcomes, with outcome rank 1 denoting the best outcome when uncertainty resolves in one-shot. Their objective probabilities are represented on the horizontal gray line. Bottom row—(2) Sequential: Panel 2(a) and 2(b) show the compounded probability weights $w_n(p) = \prod_{i=1}^{n} w(q_i)$ and the corresponding decision weights π_n when uncertainty resolves in $n = 12$ equiprobable stages, $q_i = p^{1/12}$ along a survival tree.

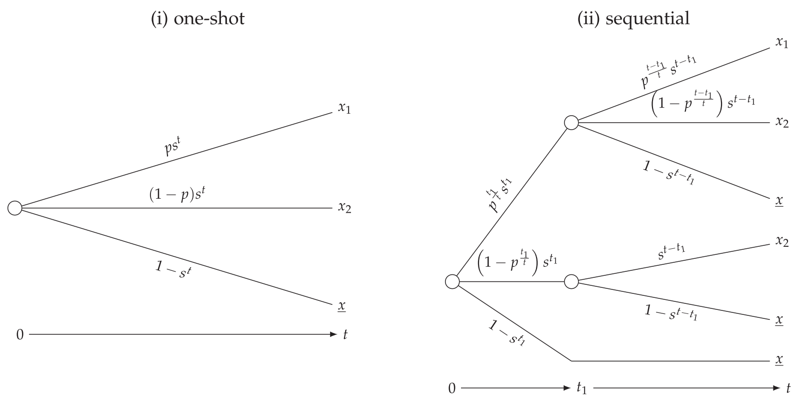

FIGURE 6. One-shot and sequential resolution of prospect and survival risk. (1) One-shot: The probability tree depicts uncertainty resolution of a prospect $(x_1, ps^t; x_2, (1-p)s^t; \bar{x}, 1-s^t)$ in one stage. (2) Sequential: The probability tree shows the sequential resolution of uncertainty of the same prospect in two stages with partial probabilities $(p^{1/t}s)^{t_1}$ and $(p^{1/t}s)^{t-t_1}$.

$$
\begin{aligned}
[V_2(\widetilde{P})]_0 &= (u(x_1)-u(x_2))w\big(p^{\frac{t_1}{t}}s^{t_1}\big)w\big(p^{\frac{t-t_1}{t}}s^{t-t_1}\big) + u(x_2)w(s^{t_1})w(s^{t-t_1}) \\
&= \left((u(x_1)-u(x_2)) \frac{w\left(p^{\frac{t_1}{t}}s^{t_1}\right) w\left(p^{\frac{t-t_1}{t}}s^{t-t_1}\right)}{w(s^{t_1})w(s^{t-t_1})} + u(x_2) \right) w(s^{t_1})w(s^{t-t_1}) \\
&= \big(\big(u(x_1)-u(x_2)\big)\,\widetilde{w}_2(p) + u(x_2)\big)\,\tilde{\rho}_2(t),
\end{aligned}
\tag{18}
$$

which yields the relationships

$$
\widetilde{w}_2(p) = \frac{w\left(p^{\frac{t_1}{t}}s^{t_1}\right) w\left(p^{\frac{t-t_1}{t}}s^{t-t_1}\right)}{w(s^{t_1})w(s^{t-t_1})}, \tag{19}
$$

and

$$
\tilde{\rho}_2(t) = w(s^{t_1})w(s^{t-t_1}), \tag{20}
$$

where $\tilde{\rho}_2(t)$ is interpreted as the discount weight attached to the allegedly certain outcome x_2. Subproportionality ensures that

$$
\widetilde{w}_2(p) = \frac{w\left(p^{\frac{t_1}{t}}s^{t_1}\right) w\left(p^{\frac{t-t_1}{t}}s^{t-t_1}\right)}{w(s^{t_1})w(s^{t-t_1})} < \frac{w(ps^t)}{w(s^t)} = \widetilde{w}(p), \tag{21}
$$

that is, under folding back observed risk tolerance is smaller than in the one-shot case, one of the main results generalized in Proposition 4 in Online Appendix B.6 where we provide a characterization of $\widetilde{w}_n$. Furthermore, total prospect value is also smaller than for one-shot resolution as both $w(ps^t)$ and $w(s^t)$ are greater than any products of probability weights of partial probabilities. Thus, the preference for one-shot resolution of uncertainty is preserved when "something may go wrong." Probability weights $\widetilde{w}$

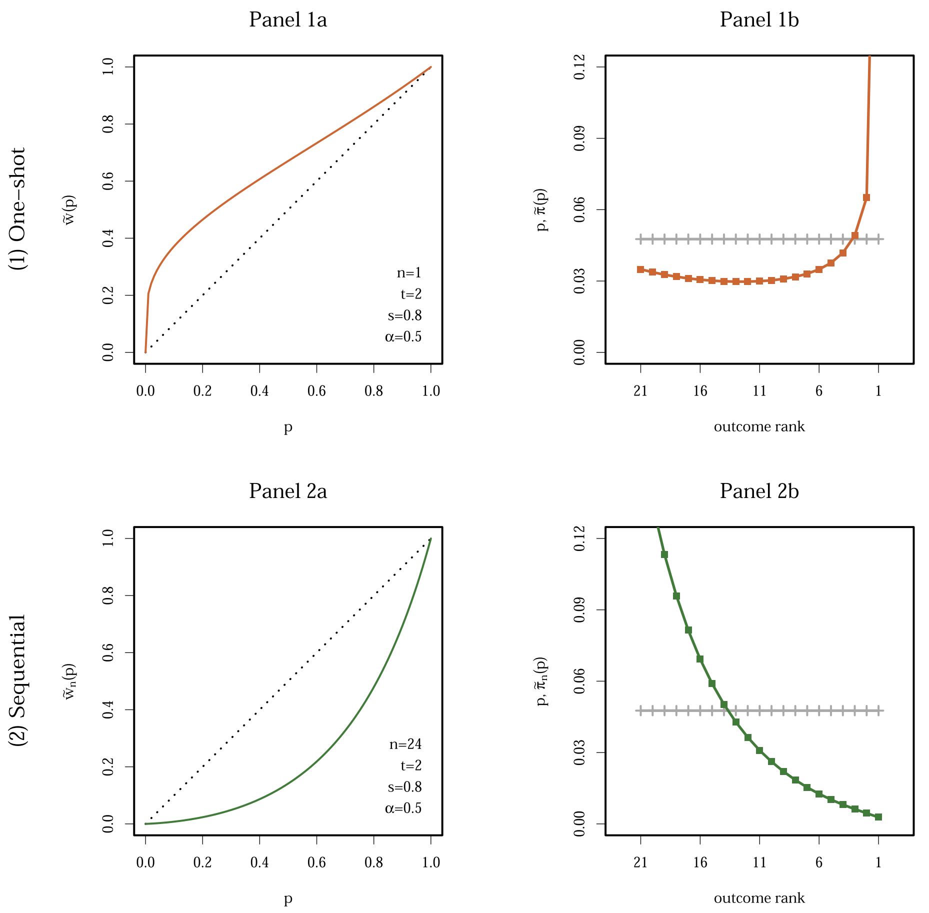

FIGURE 7. Preferences for the resolution of uncertainty with survival risk. The figure shows the impact of one-shot resolution of uncertainty versus the sequential resolution of uncertainty along a survival tree in the presence of survival risk when the prospect under consideration is delayed by $t = 2$ periods. For purposes of illustration, the curves are derived from Prelec (1998)'s compound invariant probability weighting function $w(p) = \exp(-\beta(-\ln(p))^{\alpha})$, assuming degrees of subproportionality $\alpha = 0.5$ and of convexity $\beta = 1$. Top row—(1) One-shot: The graphs show delay-dependent probability weights $\tilde{w}$ (Panel 1(a)) and their associated decision weights $\tilde{\pi}$ (Panel 1(b)) for a prospect involving 21 equiprobable outcomes, with outcome rank 1 denoting the best outcome. Their objective probabilities are represented on the horizontal gray line. Bottom row—(2) Sequential: Panel 2(a) and 2(b) show $\tilde{w}_n(p) = (w((ps^t)^{1/n})/w((s^t)^{1/n}))^n$ and the corresponding decision weights π_n when uncertainty resolves along a survival tree in $n = 24$ equiprobable stages.

and $\tilde{w}_n$ as well as their corresponding decision weights $\tilde{\pi}$ and $\tilde{\pi}_n$ are depicted in Figure 7, which show the same patterns as for the atemporal case of Figure 5, but less pronounced because delay dependence shifts the original atemporal probability weights upwards.

3.4. Prediction #4: Process Dependence of Patience

Observation #4 pertains to the finding that discount rates compounded over partial periods are higher than discount rates applied to the total period under consideration, so-called *subadditive discounting*. As we will see shortly, we can transfer all our insights for the sequential resolution of uncertainty to discounting behavior as allegedly certain future outcomes are a special case within the class of two-outcome prospects. According to our model, an allegedly certain outcome x payable at delay t is perceived as a risky future prospect $(x, s^t; \underline{x}, 1-s^t)$. Suppose now that future uncertainty resolves in two stages, first at t_1 and finally at t. Coming back to Figure 4, redefine x_2 as $\underline{x}$ and the partial probabilities as survival probabilities, $q = s^{t_1}$ and $r = s^{t-t_1}$. Subproportionaliy implies $w(s^t) > w(s^{t_1})w(s^{t-t_1})$, in other words discounting is subadditive, described as Observation #4. As before, this result holds for any number of resolution stages, and the more stages are involved the stronger the compounding effect. Moreover, discounting over equal partial periods constitutes the most aversive case.

Panel(b) of Figure 3 shows the effect of varying the number of compounding stages on observed discount rates. As predicted, discount rates increase in the number of stages. In our model, subadditive discounting is the result of decision makers' aversion to compounded probability weights and not a feature of pure time preferences themselves, as often posited in the literature.

3.5. Prediction #5: Preferences for the Timing of Uncertainty Resolution

Experimental research found a quite puzzling result: A substantial share of participants prefer uncertainty to be resolved at the payment date, even in circumstances when one would expect that it is advantageous to know the outcome of one's financial decisions as early as possible. In this section, we explore the consequences of subproportionality for the preferences for the timing of uncertainty resolution.

Figure 8 depicts two different cases of the timing of uncertainty resolution: either the prospect is played out at the payment date, corresponding to one-shot resolution and labeled "late" (Panel (i)), or the prospect is played out immediately after prospect valuation, labeled "immediate" (Panel (ii)). In the latter case, the decision maker will know the outcome right after her decision and faces only survival risk. Contrasting the resulting prospect values,

$$\begin{aligned} V_0(\widetilde{P})_{\text{late}} &= ((u(x_1) - u(x_2))\tfrac{w(ps^t)}{w(s^t)} + u(x_2))w(s^t)\rho(t) > \\ V_0(\widetilde{P})_{\text{immediate}} &= ((u(x_1) - u(x_2))w(p) + u(x_2))w(s^t)\rho(t), \end{aligned} \tag{22}$$

shows that late resolution is always preferred as $w(ps^t)/w(s^t) > w(p)$ is implied by subproportionality. Thus, if no other considerations, such as being able to make better future plans, play a role, a subproportional decision maker will exhibit a preference for late resolution of uncertainty. In fact, she will prefer resolution at t to any earlier resolution time $t_1 < t$, as shown in Online Appendix B.7.

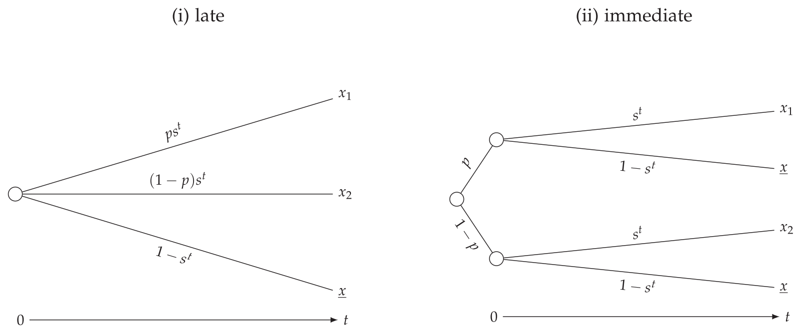

FIGURE 8. Late and immediate resolution of prospect risk. (1) Late: The probability tree depicts uncertainty resolution in one stage at the payment date t. (2) Immediate: The probability tree shows the immediate resolution of prospect risk, with survival risk resolving at t.

In our view, that subproportional risk preferences induce an intrinsic preference for late resolution of prospect risk constitutes the third important result besides delay- and process-dependence. If decision makers perceive the future as inherently risky and apply folding back, this property follows endogenously from subproportionality and does not constitute an independent preference as in the theoretical literature on resolution timing (Kreps and Porteus 1978; Chew and Epstein 1989; Grant, Kajii, and Polak 2000). Moreover, our model not only predicts a general preference for late resolution of prospect risk, it also specifically addresses skewness preferences because the effect is larger for small probabilities (see Proposition 5 in Online Appendix B.7), which cannot be handled by utility-based explanations. Additionally, this preference for late resolution of uncertainty of positively skewed prospects increases with time delay.

3.6. Prediction # 6: Risk Dependence of Patience

Researchers have been puzzled not only by delay-dependent risk tolerance and preferences with respect to resolution timing but also by other interactions between time and risk, encompassing risk-dependent discounting and diminishing immediacy: Certain outcomes tend to be discounted much more heavily than risky outcomes are. As we will show below, these findings can be naturally accommodated within our framework.

Let V_0 denote the *present value* of the prospect $P = (x_1, p; x_2, 1-p)$ delayed by t periods. Hence, for $\rho = 1$,

$$V_0 = \left((u(x_1) - u(x_2)) \frac{w(ps^t)}{w(s^t)} + u(x_2) \right) w(s^t). \tag{23}$$

Furthermore, let V_t denote the *future value* of P as of t:

$$V_t = \Big(u(x_1) - u(x_2) \Big) w(p) + u(x_2) \,. \tag{24}$$

Discounting by $w(s^t)$ yields

$$V_t w(s^t) = ((u(x_1) - u(x_2))w(p) + u(x_2))w(s^t). \tag{25}$$

According to standard discounting theory, the present value V_0 should be equal to the discounted value of V_t, namely, $V_t w(s^t)$. However, because $w(p) < w(ps^t)/w(s^t)$, actually $V_t w(s^t) < V_0$. Therefore, it seems as if the certain value V_t is discounted more heavily than the (at t equally attractive) future prospect. The difference in the valuations is not caused by different rates of time preference for risky and certain payoffs; however, but by survival risk changing the nature of the future prospect when evaluated from the point of view of the present rather than from the point of view of the future.

The same kind of risk dependence is at work when the revealed preference for a certain smaller present payoff over an allegedly certain larger later payoff decreases substantially when both payoffs are made (objectively) probabilistic, a phenomenon termed diminishing immediacy, which inspired Halevy (2008)'s work. Because of the certainty effect, the additional layer of riskiness affects the later payoff much less than the present one as it is viewed as a risky prospect already from the outset due to survival risk.

3.7. *Prediction #7: Order Dependence of Risk Tolerance*

Order dependence refers to the phenomenon that it makes a difference in which order a prospect is discounted for risk and for time. In principle, there are three different methods of establishing a decision maker's value of a prospect $P = (x_1, p; x_2, 1 - p)$ delayed by t periods: the risk-first order, the time-first order, and the direct method by which both operations are performed simultaneously.

The risk-first order assesses the certainty equivalent as of time t at the first stage and its present value at the second stage. The time-first order reverses the elicitation stages and encompasses, at the first stage, the elicitation of the present risky prospect, which is considered to be equivalent to the future one and, at the second stage, the elicitation of the certainty equivalent of this present risky prospect. The direct method, finally, elicits the present certainty equivalent of the delayed prospect in one single operation.

When the decision maker is required to state the prospect's value when discounting solely for risk, she ignores the dimension of time and reports V_t, which gets discounted to $V_t w(s^t)$ at the second stage:

$$V_t w(s^t) = ((u(x_1) - u(x_2))w(p) + u(x_2))w(s^t). \tag{26}$$

Conversely, when discounting for time first, she states the present prospect, which is equivalent to the delayed one. Discounting for risk at the second stage results in its

TABLE 3. Global parameter values.

Function	Specification	Parameter	Value
Probability weighting	Compound invariant	α: subproportionality	0.50
		β: convexity	0.95
Utility	Power	γ: curvature	0.80
Time discounting	Exponential	η: rate of time preference	0.10

Notes: The functions are specified as follows. Prelec (1998)'s compound-invariant probability weighting function: $w(p) = \exp(-\beta(-\ln(p))^{\alpha})$. Power utility function: $u(x) = x^{\gamma}$. Time discount function: $\rho(t) = \exp(-\eta t)$.

value V_0, evaluated as

$$V_0 = \left((u(x_1) - u(x_2))\frac{w(ps^t)}{w(s^t)} + u(x_2)\right)w(s^t), \tag{27}$$

which is equal to the present value elicited by the direct method.

Due to subproportionality, $w(ps^t)/w(s^t) > w(p)$. Therefore, we predict that discounting for risk first results in a lower prospect value than discounting for time first. Moreover, discounting for time first is equivalent to prospect evaluation in one single operation.

4. Quantitative Assessment

In the following, we address two issues: First, while our model is capable of qualitatively explaining all seven types of observations, it is not clear whether it also makes meaningful quantitative predictions. Second, one of the main drivers of the model is the subjective perception of future uncertainty. Can we be confident that it is actually this variable that impacts behavior?

Dealing with the first issue, the question is whether the model requires vastly different parameter values to explain the various phenomena or whether it is possible to explain them with a set of parameters within a relatively narrow and plausible range.[12] To address this question, we tie our hands and assume a fixed set of preference parameter values for (i) the utility curvature, (ii) the degrees of subproportionality and convexity of probability weights, and (iii) the rate of pure time preference, as specified in Table 3. These parameter values are suggested by typical estimates in the literature (see, e.g., Abdellaoui, Diecidue, and Öncüler (2011b); Epper, Fehr-Duda, and Bruhin (2011); Fehr-Duda and Epper (2012)). We estimate the annual survival probabilities s^* by minimizing the sum of squares of the deviations of the actual observed quantities and the values predicted by our model given the fixed preference parameters. That is, s^* is the only free parameter to be estimated. To assess the accuracy of the predictions of our model in the different experimental

12. We thank an anonymous referee for proposing this calibration exercise.

conditions (e.g., different delays, probabilities, resolution frequencies, or timings), we contrast the observed quantities reported in the respective publication with those predicted by our model using the estimated survival probability and fixed preference parameter values.

Ideally, for a given participant sample at a given point in time, and a given elicitation method, the estimated value of s should be rather similar across experiments because in this case, the participants would have little reason to reveal different degrees of subjective uncertainty that "something may go wrong." However, the seven regularities we discussed have been documented at different points in time, with different elicitation methods, and with rather different participant samples—French, Swiss, Swedish, and US participants. Table C.1 in Online Appendix C summarizes the experimental studies we used for our task. Therefore, the best we can hope for is that the estimated value of s is roughly in a similar ballpark across the different experiments. In addition, because all studies have been conducted with university students in Western countries with well-developed property rights, the estimated value of s should not be unreasonably low (e.g., below 0.5 or 0.6 p.a.). As we will see below, our quantitative estimates nicely confirm these expectations. The typical value of the survival probability across experiments is around 0.9 and never below 0.825. Thus, all seven phenomena can be quantitatively explained with a plausible and identical set of preference parameter values and a narrow and plausible range of survival probabilities.

Regarding the second issue, it is not obvious that the estimated level of survival probability s actually captures people's perceived uncertainty. To underpin the credibility of our approach, we present data on participants' perceptions and relate them to the magnitudes of s estimated at the individual level. It turns out that people who reported some uncertainty with respect to obtaining future payments exhibit significantly lower levels of survival probability, corroborating our approach.

4.1. Observation #1: Probability Weights Increasing with Delay

To demonstrate the quantitative implications of our approach, we proceed as follows. According to our framework, the driver of risk tolerance increasing with delay are delay-dependent probability weights. Delay-dependent risk tolerance was observed in many experiments, but only a very few provide estimates of suitable probability weights. One particularly useful example is Abdellaoui et al. (2011a)'s investigation of the source dependence of uncertainty attitudes. Their experiment also involved pure risk, that is, given objective probabilities, as a special source for which uncertainty resolved at the payment date 3 months after the experimental sessions. Abdellaoui et al. (2011a) assume a Prelec (1998) compound-invariant probability weighting function and report $\hat{\alpha} = 0.67$ and $\hat{\beta} = 0.76$ for the delayed weights $w_{t=3}(p)$ (see their Figure 9 on page 713). Now, what is the level of survival probability s such that their delayed weights $w_{t=3}(p)$ can be interpreted as $\widetilde{w}(p)$ based on the atemporal weights $w_{t=0}(p)$ generated by our global parameter values? We estimate s by minimizing the sum of squares of the difference between $w_{t=3}$ and $\widetilde{w}$. This exercise yields an estimated s^* of 0.825 p.a., which we deem a very plausible number. In other words, subjects behaved

FIGURE 9. Observed versus predicted weighting functions.

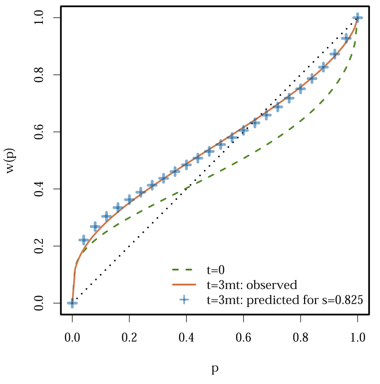

TABLE 4. Decision weights.

Probability	Observed	Predicted
0.125	0.289	0.308
0.250	0.388	0.395
0.500	0.552	0.544
0.750	0.719	0.710
0.875	0.821	0.821

INCREASING RISK TOLERANCE. **Figure (9)** The green dashed curve corresponds to the atemporal probability weighting function, $w_{t=0}$, generated by our global parameter values. The red solid curve depicts the probability weighting function estimated by Abdellaoui et al. (2011a) for uncertainty resolution in 3 months, $w_{t=3}$ (see Panel (b) in their Figure 9 on page 713). The blue crossed curve results from the global parameter values of Table 3 and a survival probability of $s^* = 0.825$.

Table (4) For each of the five probabilities in Abdellaoui et al. (2011a), the table contrasts observed decision weights of the better outcome with decision weights predicted by our model with global parameter values and $s^* = 0.825$. The observed weights are computed using the probability weighting function estimated by Abdellaoui et al. (2011a).

as if they thought outcomes payable in 1 year would actually materialize with a 82.5% chance. As one can see in Figure 9, the curve of $\widetilde{w}$ for this level of s^* closely matches the actual reported curve $w_{t=3}$.

4.2. Observation #2: Hyperbolic Discounting

Epper, Fehr-Duda, and Bruhin (2011) elicited both time preferences and risk preferences of a student sample.[13] Comparing the average annualized discount rates observed for a 2-month delay and a 4-month delay shows the usual picture: they decline from 0.368 to 0.299 when the delay increases (all these numbers can be found in the first column of their Table 2 on page 183 of the paper). Assuming that the discount rates are generated by the theoretical discount weights $w(s^t)\exp(-\eta t)$, we estimate s by minimizing the sum of squared deviations between observations and model predictions. This procedure yields $s^* = 0.947$, resulting in prediced discount rates of 0.372 for the 2-month delay, and 0.293 for the 4-month delay, shown in Table 5, which are very close to the observed values.

13. Based on the risk taking data, Epper, Fehr-Duda, and Bruhin (2011) estimated the mean Prelec $\alpha = 0.505$ and the mean $\beta = 0.974$, which lie very close to our global parameter values.

TABLE 5. Hyperbolic discounting—observed versus predicted discount rates.

Delay	Observed	Predicted
2 months	0.368	0.372
4 months	0.299	0.293

Notes: The table lists observed and predicted annualized discount rates for the two different time delays in Epper, Fehr-Duda, and Bruhin (2011). The observed rates can be found in the first column of Epper, Fehr-Duda, and Bruhin (2011)'s Table 2 (p. 183). The predicted rates result from our model with the global parameter values of Table 3 and $s^* = 0.947$.

TABLE 6. Process dependence—observed versus predicted certainty equivalents.

Prospect	Condition	Observed	Predicted
$(50, 1/12; 0)$	One-shot	9.910	11.184
	Sequential	9.250	7.182
$(50, 1/2; 0)$	One-shot	22.650	22.671
	Sequential	20.720	19.052
$(50, 11/12; 0)$	One-shot	37.740	37.781
	Sequential	34.720	35.435

Notes: The table lists certainty equivalents documented in Table 2 on page 1310 of Abdellaoui, Klibanoff, and Placido (2015) for one-shot and sequential resolution (their "CRG" condition). The predictions are obtained with the global parameter values of Table 3.

4.3. Observation #3: Preference for One-Shot Resolution of Uncertainty

To the best of our knowledge, the process dependence of risk taking behavior has not been investigated experimentally in situations when there is actually a substantial time delay present. Experimental tasks are typically based on one-stage and distributionally equivalent multi-stage prospects that are resolved almost immediately. In other words, the survival probability s is irrelevant in such situations. Thus, we will illustrate an atemporal version of the preference for one-shot resolution of uncertainty over sequential resolution by (i) documenting that the predicted certainty equivalents of one-shot resolved prospects are higher than those of sequentially resolved prospects, and (ii) by comparing the actually observed certainty equivalents with the estimated certainty equivalents resulting from our assumptions.

Abdellaoui, Klibanoff, and Placido (2015) report mean certainty equivalents for simple prospects of the form $(50, p; 0, 1 - p)$ (their Table 2 on page 1310) that are resolved in one stage or in two stages. Table 6 shows that one-shot certainty equivalents are always higher than sequential ones, and the estimated values, based on our global parameter values, are reasonably close to the observed ones, particularly for the probabilities $1/2$ and $11/12$. For $p = 1/12$, the model overestimates the difference between one-shot and sequential values.[14]

14. Regarding the other features of our model, event commutativity and aversion to equiprobable stages, the evidence so far is mixed. For a review, see Fan, Budescu, and Diecidue (2019).

TABLE 7. Subadditive dscounting—observed versus predicted discount fractions.

Discount Fraction	Observed	Predicted
$f(0,2)$	0.927	0.893
$f(2,4)$	0.941	0.893
$f(0,4)$	0.886	0.852
$f(0,2)f(2,4)$	0.872	0.797

Notes: The table lists discount fractions for various payment dates and the relevant product. Observed values are derived from the values shown in Table 2 of Epper, Fehr-Duda, and Bruhin (2011) (p. 183). Predictions are derived by our model with the global parameter values of Table 3 and $s^* = 0.947$.

4.4. *Observation #4: Subadditive Discounting*

To illustrate the quantitative implications of our model, we examine the discounting data of Epper, Fehr-Duda, and Bruhin (2011) again. In the experiment, future equivalents FEs of a fixed sooner amount of CHF 60 were elicited for various time delays. We define the observed *discount fraction* as

$$f(t_i, t_j) = \frac{60}{\text{FE}},$$

where t_i is the payment date for the sooner amount 60 and t_j the payment date for the later amount FE (Read 2001). If the product $f(t_1,t_2)f(t_2,t_3)$ is smaller than the discount fraction over the total period, $f(t_1,t_3)$, then discounting is subadditive. According to our model, indifference between sooner and later payments is given by

$$u(60)w(s^{t_1})\exp(-\eta t_1) = u(\text{FE})w\left(s^{t_2-t_1}\right)w\left(s^{t_1}\right)\exp(-\eta t_2).$$

Assuming power utility with parameter γ, the discount fraction equals to

$$f(t_1,t_2) = \frac{60}{\text{FE}} = \left(\frac{w(s^{t_2-t_1})\exp(-\eta t_2)}{\exp(-\eta t_1)}\right)^{\frac{1}{\gamma}}.$$

Given the estimated survival probability derived for the same data set of Observation #2, $s^* = 0.947$, the following predictions for the discount fractions result, listed in Table 7. Both the observed mean discount fractions and the predicted ones clearly exhibit subadditivity, with predictions fitting fairly well.

4.5. *Observation #5: Preference for Late Resolution of Uncertainty*

Arai (1997) measured strength of preference (SOP) toward resolution timing for delayed prospects that varied by outcome probability and time delay. In this case, we do not have present certainty equivalents at our disposal but have to rationalize SOP values. We report Arai (1997)'s findings on the prospect $(5000, p; 0, 1-p)$ listed in Table 1 on page 20 of his paper. SOP was measured on a scale divided into 30 equal intervals, with SOP $= 0$ denoting strong preference for immediate resolution and SOP $= 30$ denoting strong preference for late resolution. Thus, SOP $= 15$ signals indifference between immediate and late resolution of uncertainty.

TABLE 8. Resolution timing—$W(p,t)$ and SOP.

	$t = 1/4$		$t = 2$		$t = 10$	
p	$W(p,t)$	SOP	$W(p,t)$	SOP	$W(p,t)$	SOP
0.05	1.16	16.4	1.46	17.0	2.03	17.8
0.35	1.15	15.6	1.41	16.5	1.77	18.2
0.65	1.14	12.4	1.35	14.4	1.55	17.2
0.95	1.11	12.3	1.18	13.9	1.21	16.9

Notes: The table shows wedges $W(p,t) = w(ps^t)/w(p)w(s^t)$ predicted by our model with global parameter values of Table 3 and $s = 0.9$ and observed strength of preferences values reported in Arai (1997) (Table 1 on page 20).

Arai (1997) finds a very distinct pattern of SOP depending on time delay and probability: the smaller the probability and the longer the time delay, the stronger the preference for late resolution. Our task is to predict the patterns observed by Arai (1997). For this purpose, we examine the wedge $W(p,t) := w(ps^t)/(w(p)w(s^t))$, which measures the decision weight for late resolution relative to the decision weight for immediate resolution of uncertainty. We hypothesize that it is more likely to observe SOP > 15 in favor of late resolution for greater values of the wedge $W(p,t)$. We calculate $W(p,t)$ by assuming our global parameter values and survival probability $s = 0.9$, which lies in the range of s^* found for the other phenomena (see Table 12 below.)

Table 8 shows a totally consistent picture, $W(p,t)$ is predicted to decrease in p and increase in delay t, capturing the patterns in the observed SOP measures. The Spearman rank correlation coefficient between SOP and $W(p,t)$ amounts to 0.902, which we deem an exceptionally high value.

4.6. Observation #6: Diminishing Immediacy

In their experiments, Weber and Chapman (2005) investigated whether delaying an outcome is equivalent to making it risky. In one of these experiments, participants' present certainty equivalents for delayed prospects were elicited through a series of choices using a bisection algorithm. A total of 124 participants supplied useful responses in the immediacy task, which involved hypothetical amounts of \$100 and \$110. These amounts were due either immediately or with various time delays, and were supposedly certain or risky materializing with a probability of $p = 0.5$.

Working with our global parameters, we estimated the survival probability that minimizes the sum of squares of differences between observed and predicted values. This exercise resulted in an estimate of $s^* = 0.872$, again a very reasonable number.

Table 9 contrasts observed present certainty equivalents[15] with predicted ones. Generally, we are able to produce quite a good match between observed and

15. Values for present certain \$100 were not elicited.

TABLE 9. Risk-dependent discounting—observed versus predicted present certainty equivalents.

Delay	Amount	Probability	Observed	Predicted
0	100	1.0	100.00	100.00
		0.5	38.32	37.21
4	110	1.0	70.52	81.86
		0.5	35.46	38.02
26	100	1.0	41.11	39.94
		0.5	23.34	23.40
30	110	1.0	47.85	40.16
		0.5	23.75	24.03

Notes: The table lists present certainty equivalents reported in Weber and Chapman (2005); Table 5 (p. 111). The predicted present certainty equivalents are obtained using our model with the global parameter values of Table 3 and $s^* = 0.872$.

TABLE 10. Diminishing immediacy—predicted discount weights.

Delay t	Amount	Probability	Discount weight
4	110	1.0	92.4%
		0.5	94.3%
26	100	1.0	59.8%
		0.5	69.0%
30	110	1.0	55.3%
		0.5	65.3%

Notes: The table lists predicted discount weights for the different delayed prospects in Weber and Chapman (2005) based on the global parameter values in Table 3 and $s^* = 0.872$.

estimated values, only the present value of 110 materializing in 4 months is overstated by the model, that is, participants discounted 110 much more heavily than estimated. According to our model, an allegedly certain outcome payable at delay t, (x, t), is evaluated as $u(x)w(s^t)\exp(-\eta t)$. Its risky counterpart is evaluated as $u(x)w(ps^t)\exp(-\eta t)$. Their corresponding non-delayed values amount to $u(x)$ and $u(x)w(p)$, respectively, implying the discount weights $w(s^t) < w(ps^t)/w(p)$ for the certain and risky outcomes. Comparing the entries for $p = 1$ and $p = 0.5$ for the various delays in Table 10 clearly shows a greater loss in value for allegedly certain outcomes than for risky ones.

4.7. *Observation #7: Preference for Time-First Order of Prospect Valuation*

In their study on order dependence, Öncüler and Onay (2009) found the following pattern: While valuations of delayed risky prospects resulting from the time-risk order ("TR", discounting for time first and for risk thereafter) and the direct method ("D", both operations performed simultaneously) are not statistically distinguishable from each other, risk-time evaluations ("RT", discounting for risk first and for time thereafter) are significantly lower than the ones obtained from the other two methods.

TABLE 11. Order dependence—observed versus predicted present certainty equivalents.

Probability	Condition	Observed	Predicted
0.5	RT	35.94	34.09
	TR	39.83	37.06
	D	39.60	37.06
0.3	RT	22.07	24.89
	TR	24.44	27.09
	D	24.14	27.09

Notes: The table shows observed present certainty equivalents reported in Öncüler and Onay (2009), Table 1 on page 285. The predictions are obtained by our model using the global parameter values of Table 3 and $s^* = 0.937$.

TABLE 12. Summary: estimated survival probabilities s^*.

Observation #	Output variable	s^* p.a.	Remark
1	Probability weights	0.825	
2	Discount rates	0.947	
3	Certainty equivalents	–	Not relevant
4	Discount fractions	0.947	same as in #2
5	Correlation with preference strength	0.900	assumed
6	Present certainty equivalents	0.872	
7	Present certainty equivalents	0.937	

Notes: The table lists estimated survival probabilities for each observation (see the remarks for exceptions). Survival probabilities are estimated by minimizing the sum of square deviations between observed and predicted output variables based on the global parameter values of Table 3.

Here, we proceeded as before, we minimized the sum of squared deviations between observations and predicted magnitudes based on our global parameter values, which resulted in an optimal survival probability $s^* = 0.937$. We report the observed and predicted present certainty equivalents for the three elicitation methods in Table 11. Predictions match observations quite well.

4.8. Summary of Quantitative Assessment

The quantitative assessments conducted in this section were based on the same set of preferences parameter values. We deliberately tied our hands for the quantitative predictions by assuming a plausible set of parameter values suggested by the literature. In this way, we avoid arbitrary degrees of freedom in accommodating the data and enable a judgment to what extent our approach indeed facilitates a unifying explanation of a diverse set of phenomena. Table 12 shows that the objects of interest that needed to be assessed to explain the experimental findings were quite varied—ranging from probability weights to discount rates, from discount fractions to (present) certainty equivalents. Our quantitative analysis suggests that the model generally fits the observations well. Furthermore, as Table 12 reveals, we find that observed behavior is consistent with plausible values of an annual survival probability in the range of

0.825–0.947. In view of the fact that the data were elicited from different participant samples in different countries and at different points in time, we deem this a remarkably narrow and plausible range of values for survival probability.

Recently, a team of researchers conducted an experiment on risk taking and time discounting with the explicit objective of estimating the probability of prospect survival s (Islam, Diecidue, and Hardardottir 2022). Not only are their results consistent with our assumptions on global preference parameter values but also, and most importantly, they come up with an average estimate of s of 0.934, which lies nicely within the range of our assessments. Thus, Islam, Diecidue, and Hardardottir (2022) present an independent measure of s that corroborates our findings.

4.9. The Perception of Future Uncertainty

While the quantitative assessments in the previous section renders plausible values of survival probabilities, it is not a priori clear that s actually captures perceptions of future uncertainty or something else. In order to address this issue, we use new data that allows us to tap into these subjective perceptions. We use data on time discounting and risk taking of 282 individuals recruited from the Swiss German speaking population, by a professional survey institute. Details of the experimental design and procedures as well as the estimation strategy are set out in the Online Appendix D.

To measure time discounting, we elicited 28 sooner equivalents with a maximum payment of CHF 80 and a maximum delay of 8 months. The risk taking tasks involved the elicitation of 20 certainty equivalents of binary lotteries with outcomes in the same ranges as the delayed ones. Furthermore, we asked the participants about their perceptions of future uncertainty in a questionnaire following the choice tasks. We posed the following question: “Which of the following factors influenced your choices between sooner and later payments?” There were four items pertaining to potential sources of future uncertainty:

1. For some reason it may be impossible for me to obtain the money.
2. It is possible that the money will not be delivered.[16]
3. The survey organizers are not trustworthy.
4. Other factors that cannot be influenced.

Participants had to report their degrees of agreement with respect to these statements by five different response categories: “clearly yes”, “rather yes”, “do not know”, “rather not”, and “not at all”. We constructed a binary variable UNCERTAINTY from the respective responses in the following way: Whenever a participant responded with “clearly yes” or “rather yes” to any of the four items, UNCERTAINTY was assigned a value of 1, 0 otherwise. 24.1% of the participants responded in the affirmative and, consequently, UNCERTAINTY was assigned a value

16. Note that experimental earnings were sent by mail.

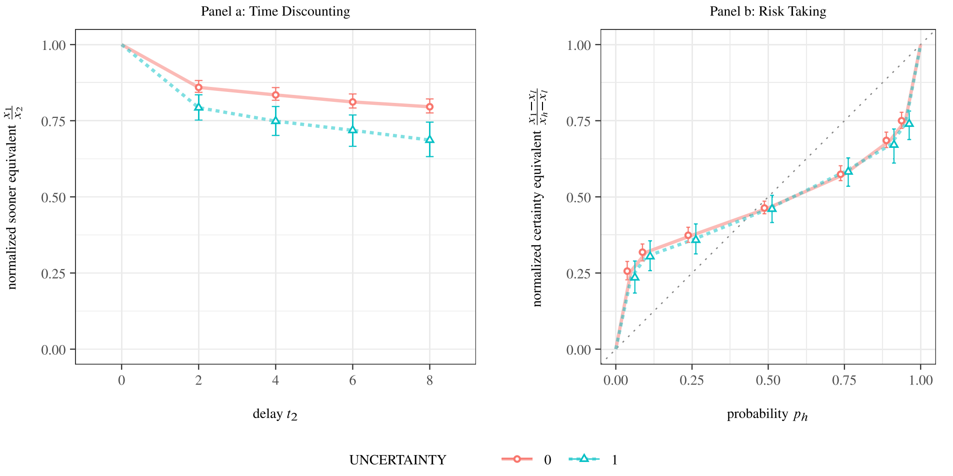

FIGURE 10. Descriptive results: The dependence of time discounting and risk taking on uncertainty perception. Panel (a): The figure plots mean normalized sooner equivalents against the delay t_2 in months. There were four different delays, $t_2 \in \{2, 4, 6, 8\}$ months. The circles and triangles indicate the mean normalized sooner equivalents. The whiskers depict the 95% confidence intervals around the means. The confidence intervals were constructed using the bootstrap method with 1,000 replications clustered at the individual level. Panel (b): The figure plots mean normalized certainty equivalents against the probability of obtaining the better lottery outcome p_h. There were seven different probability levels, $p_h \in \{0.5, 0.1, 0.25, 0.5, 0.75, 0.9, 0.95\}$. The circles and triangles indicate the mean normalized certainty equivalents. The confidence intervals were again constructed using the bootstrap method. The means are slightly horizontally dodged for better visibility.

of 1, whereas 75.9% of the participants did not identify any of the potential factors as relevant for their intertemporal decisions. Consequently, we label these two different groups “uncertain participants” and “certain participants”, respectively.

In order to find out whether the uncertain participants differed from the certain ones with respect to their time discounting and risk taking behaviors, we analyzed the raw data on sooner equivalents and certainty equivalents for each group separately. First, we normalized the respective magnitudes to make them comparable: We elicited the sooner equivalent x_1 at $t_1 = 0$ for a given fixed delayed amount x_2 at t_2 and divided this value by x_2, which resulted in a *normalized sooner equivalent* x_1/x_2. We used an equivalent approach for normalizing the certainty equivalent y for a given prospect $(x_h, p_h; x_l, 1 - p_h)$ rendering the *normalized certainty equivalent* $(y - x_l)/(x_h - x_l)$. The results of this exercise are displayed in Figure 10. Panel (a) shows, conditional on the value of UNCERTAINTY, the normalized sooner equivalents for different lengths of delay t_2, which can be interpreted as cash discount weights. Clearly, sooner equivalents of uncertain participants differ significantly from those of certain participants: Participants who had voiced concerns about future payments discounted future amounts much more heavily with discount weights declining more steeply.

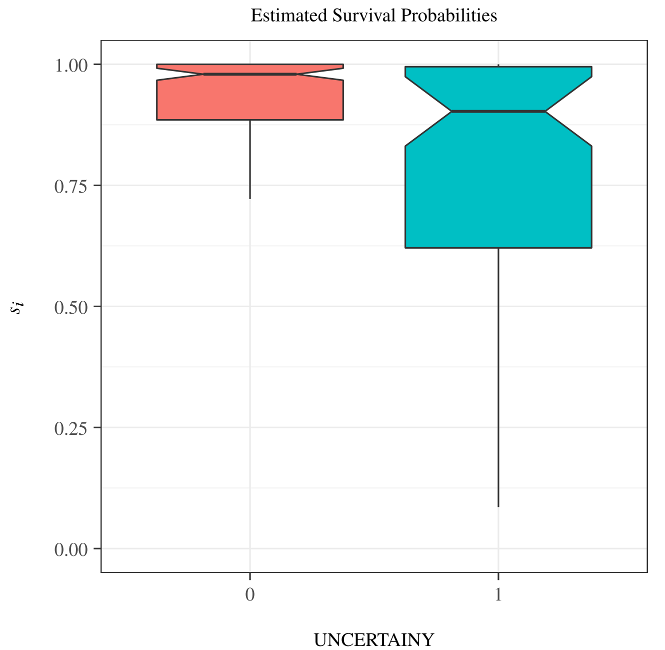

FIGURE 11. Survival probability and the perception of future uncertainty. The figure shows information on the distributions of the estimated individual survival probabilities s_i conditional on the values of the binary variable UNCERTAINTY indicating uncertainty perception. The height of the colored boxes indicate the interquartile ranges (IQR). The horizontal bold lines dissecting these colored areas indicate the medians. The notches, the widths of the indentations around the medians, give roughly 95% confidence intervals for comparing the medians, with median $\pm$ 1.58 $\times$ IQR/$\sqrt{n}$ being their lower/upper end. The vertical lines, the whiskers, indicate the smallest/largest values at most 1.5 $\times$ IQR from the boxes' boundaries.

Panel (b) shows the normalized certainty equivalents for various levels of probability p_h. The resulting curve can be interpreted as a probability weighting function in the cash domain. Unlike in the discounting case, certainty equivalents do not differ between the two groups. Therefore, it is safe to assume that risk preference parameters do not differ between uncertain and certain participants.

The question now arises whether there is a significant relationship between perceived uncertainty, as measured by our binary variable, and the probability of prospect survival, as estimated according to our modeling approach. According to the model, participants with UNCERTAINTY=1 should exhibit lower levels of s than participants with UNCERTAINTY=0. To answer this question, we proceeded as follows: We estimated a model that allows for individual differences in survival probabilities, but kept the preference parameters constant at the levels defined in Table 3. It turns out that the distributions of the estimated individual survival probabilities differ substantially between certain and uncertain participants. The box plots in Figure 11 display the respective results. Confirming our conjecture, the mean of the estimated survival probabilities s lies much closer to 1, namely, at 0.873, for the participants who reported that uncertainty did not impact their decisions, whereas the mean for the uncertain participants amounts to only 0.770. To complete our analysis, we also estimated a model allowing for individual heterogeneity in risk and time preference parameters. As shown in Online Appendix D, the magnitudes of the estimated survival probabilities change somewhat, but there are still clear and substantial differences between certain and uncertain participants. Thus, we can be confident that the estimated survival probabilities do indeed capture aspects of individual perceptions of future uncertainty.

5. Experimental Findings and Related Literature

In the following section, we present the experimental literature and previous explanations of the observed effects in more detail. As will become evident, so far none of the contributions provided an integrative view of all the seven phenomena. A comprehensive list of the relevant experimental papers is included in Online Appendix A. We also discuss a number of additional papers that are related to our work in Section 5.2.

5.1. Literature Related to the Seven Observations

Turning to the first behavioral phenomenon in our list in Table 1, delay dependence of risk taking behavior has been documented by a range of papers that do not distinguish between effects of delay on utility and probability weights (Jones and Johnson 1973; Shelley 1994; Ahlbrecht and Weber 1997; Sagristano, Trope, and Liberman 2002; Noussair and Wu 2006; Coble and Lusk 2010). That, in fact, probability weights react to delay, rather than the utility function, was shown experimentally by Abdellaoui, Diecidue, and Öncüler (2011b). They conducted a carefully designed experiment eliciting probability weights for both present and delayed prospects. Their results provide support for our approach as the probability weights of the best possible outcome, when delayed, are significantly greater than their non-delayed counterparts, both in the aggregate as well as for the majority of the individuals. In their study on ambiguity attitudes, Abdellaoui et al. (2011a) show estimates of a probability weighting function derived from choices over prospects delayed by 3 months, which we used to assess the quality of our predictions. This function is also much more elevated than typical atemporal estimates are, that is, the curve lies above a typical atemporal one; see Figure 9.

To the best of our knowledge, there is only one theoretical contribution that derives the delay dependence of risk tolerance from a set of axioms. Baucells and Heukamp (2012) analyze the case of simple prospects $[(x, p; 0, 1-p)]_t$ that pay x with probability p at time t and 0 otherwise. They derive risk tolerance increasing with delay in the following way. Their fundamental axiom links risk taking and time discounting by direct assumptions on how people trade-off delays in future outcomes against reductions in the probability with which these outcomes occur. Additionally, they assume the presence of common ratio effects, which is equivalent to subproportionality. As already set out in Section 3.1, the authors derive time-dependent probability weights $g(p;t) = g(p\exp(-r_x t))$ that, unlike our representation, clearly decrease with delay t. Thus, they need another crucial assumption to predict risk tolerance increasing with delay: They make the probability-time trade-off depend on outcome magnitude—the probability that renders an early prospect equally attractive as a prospect with a fixed additional delay declines with outcome magnitude, in other words r_x has to decline in x. Their approach also predicts hyperbolic discounting (Observation #2) (for this result, the common ratio effect has to hold as well as decreasing elasticity of the utility function) and risk dependence of patience (Observation #6), which is a direct consequence of the probability-time trade-off under subproportionality.

It is well known by now that delay dependence is also manifest in discounting behavior, which constitutes empirical Observation #2. There is abundant evidence that many people exhibit decreasing impatience, that is, their discount rates are not constant but decline with the length of delay (among many others Benzion, Rapoport, and Yagil (1989); Loewenstein and Thaler (1989); Ainslie (1991); Halevy (2015)). This regularity has triggered a large literature on hyperbolic and quasi-hyperbolic time preferences (e.g., Laibson (1997), for reviews; see Frederick, Loewenstein, and O'Donoghue (2002) and Ericson and Laibson (2019)). As already mentioned in the introduction, most closely related to our approach is the string of papers following Halevy (2008), which derives hyperbolic discounting from the same mechanism that we employ, namely, a combination of future uncertainty with subproportional probability weighting. The subsequent contributions by Saito (2011) and Chakraborty, Halevy, and Saito (2020) are concerned with establishing a two-way relationship between subproportional probability weights and hyperbolic discounting. The final paper in this series clarifies that subproportionality both implies and is implied by hyperbolic discounting in the domain of single temporal prospects in continuous time, the objects of our model. For consumption streams in discrete time, Halevy (2008)'s original topic, subproportionality still implies hyperbolic discounting, but the reverse direction requires more involved conditions, however.

Intertemporal choice is the objective of Kőszegi and Szeidl (2013)'s model of focusing. By explicitly taking into account attributes of the decision context, their model of attention is able to predict when people exhibit present or future bias. Our approach is able to generate future bias as well, if the decision maker is prone to a reverse common ratio effect (i.e., if the probability weighting function is supraproportional). Gabaix and Laibson (2022) propose a yet different approach to time discounting. They derive hyperbolic discounting from the assumption that decision makers obtain unbiased but noisy simulations of future utilities. Both the source and the nature of uncertainty differ between their approach and ours: In their model, uncertainty captures the fact that the decision maker does not know the actual future utility she will experience. Simulation noise makes future utility more risky (in terms of second-order stochastic dominance). In contrast, we model the fact that "something may go wrong," which adds a downside risk to future prospects (in terms of first-order stochastic dominance). Moreover, Gabaix and Laibson (2022) do not study the interaction of risk and time preferences.

Epper, Fehr-Duda, and Bruhin (2011) provide experimental evidence that common ratio violations and non-constant discounting are actually exhibited by the same people. Using the decline of discount rates as a measure of decreasing impatience, Epper, Fehr-Duda, and Bruhin (2011) show that participants' departures from linear probability weighting are indeed highly significantly correlated with the strength of the decrease in discount rates. In fact, the only variable associated with decreasing discount rates turns out to be the degree of subproportionality of probability weights, which explains a large percentage of the variation in the extent of the decline, whereas observable individual characteristics, such as gender, age, experience with investment decisions, and cognitive abilities are not significantly correlated with the degree of non-constant discounting. Thus, their paper provides the first evidence that subproportionality is indeed an important driver of discounting behavior.

Observations #3 and #4 concern the process dependence of risk taking and time discounting behavior. In the domain of risk, the prevalent finding is that, on average, subjects do not reduce compound probabilities according to the rules of probability calculus. For example, Aydogan, Bleichrodt, and Gao (2016) show that for their participants, the reduction principle is clearly violated at the aggregate level even though 60% of subjects behave in accordance with reduction. The aggregate result is driven by a minority of participants who depart strongly from reduction—in this case in the direction of a preference for sequential resolution. The authors attribute this finding to the utility of gambling. However, there is also abundant experimental evidence that the value of a compound lottery is smaller than the value of the equivalent single-stage lottery, for example, Chung, von Winterfeldt, and Luce (1994), Budescu and Fischer (2001), Abdellaoui, Klibanoff, and Placido (2015), and Fan, Budescu, and Diecidue (2019) to name a few. It seems to be the case that the framing of the experimental tasks plays a role whether one finds a preference or an aversion to compound risks (Nielsen 2020).

A related category of results concerns investment games (Gneezy and Potters 1997; Thaler et al. 1997; Bellemare et al. 2005; Gneezy, Kapteyn, and Potters 2003; Haigh and List 2005). The general finding is that people tend to invest less conservatively, that is, they take on more risk, when they are informed about the outcomes of their decisions only infrequently. This finding is often interpreted as a manifestation of *myopic loss aversion*, a term coined by Benartzi and Thaler (1995). In this context, myopia is defined as narrow framing of decision situations, which focuses on short-term consequences rather than on long-term ones. Loss aversion, one of the key constituents of prospect theory, describes people's tendency to be more sensitive to losses than to gains. According to this interpretation, if people evaluate their portfolios frequently, the probability of observing a loss is much greater than if they do so infrequently.[17]

17. In these experiments, subjects evaluate *sequences* of identical two-outcome lotteries over several periods where the range of potential outcomes increases with the number of periods. As we noted in Section 3.3, subproportionality does not deliver clear predictions for this class of prospects. However,

Whatever the specific experimental context, however, all these experiments share the feature that time delays were negligible. Tests of process dependence in genuinely temporal settings are still lacking.

Process dependence of risk taking was theoretically analyzed in the seminal contributions of Segal who deals with the evaluation of two-stage prospects in the domain of RDU (Segal 1987a, b, 1990). Dillenberger (2010) provides a necessary and sufficient condition for preferences for one-shot resolution of uncertainty, which holds, for example, in Gul (1991)'s theory of disappointment aversion, but not generally in RDU. However, we show in Online Appendix E.2 that this preference condition also applies to the class of resolution processes studied here.

With respect to Observation #4, process dependence has also been observed in the domain of time discounting: Discount rates applied to a particular delay are higher when the delay is divided into subintervals than when it is left undivided (Read 2001; Read and Roelofsma 2003; Ebert and Prelec 2007; Epper, Fehr-Duda, and Bruhin 2009; Dohmen et al. 2017). This regularity of subadditive discounting has usually been interpreted as a manifestation of (pure) time preferences (Read 2001).

Observation #5 refers to the effect of the timing of uncertainty resolution on risk taking behavior. Several experimental studies investigated people's intrinsic preferences for resolution timing. The general finding is that there are varying percentages of people with preference for early resolution, preference for late resolution and timing indifference (Nielsen 2020). Often, the percentage of people with a preference for late resolution is quite sizable (Chew and Ho 1994; Ahlbrecht and Weber 1996; Arai 1997; Lovallo and Kahneman 2000; Eliaz and Schotter 2007; von Gaudecker, van Soest, and Wengström 2011; Ganguly and Tasoff 2017).[18] This finding is actually quite surprising, at least for situations where real money is at stake. Knowing early how much income to expect should always be advantageous for adapting one's consumption plans even though one might not be able to spend the money immediately.

An intrinsic preference for resolution timing cannot be accommodated by EUT but is usually modeled by an additional preference parameter (Kreps and Porteus 1978; Chew and Epstein 1989; Grant, Kajii, and Polak 2000). What these models cannot capture, however, is the probability dependence of timing preferences, as found by Arai (1997), for example. Epstein and Kopylov (2007)'s and Epstein (2008)'s axiomatic papers analyze resolution timing as well. According to their approach, decision makers may become more pessimistic as payoff time approaches, either due to changes in beliefs or anticipatory feelings (see also Caplin and Leahy (2001)).

Langer and Weber (2005) show that the same is true for myopic loss aversion—for specific risk profiles, myopia will not decrease but increase the attractiveness of a sequence. Blavatskyy and Pogrebna (2010) also contest the validity of the myopic loss aversion hypothesis.

18. Epstein and Zin (1991) also find a preference for late resolution of uncertainty in market data on U.S. consumption and asset returns. In line with our predictions, preference for late resolution seems to be particularly pronounced for positively skewed distributions, that is, for prospects with small probabilities of the best outcome, and increases with time delay—a prediction that is a distinguishing feature of our model.

Observation #6 pertains to a number of experimental studies that report systematic effects of risk on discounting behavior: Discount rates for certain future payoffs tend to be higher than discount rates for risky future payoffs (Stevenson 1992; Ahlbrecht and Weber 1997; Abdellaoui et al. 2018). Risk-dependent discounting is also evident in diminishing immediacy: People's preference for present certain outcomes over delayed ones, immediacy, weakens drastically when the outcomes become risky—they behave as if they discounted the risky reward less heavily than the original certain one (Keren and Roelofsma 1995; Weber and Chapman 2005; Baucells and Heukamp 2010). This evidence motivated Halevy (2008)'s conjecture that future uncertainty might be the driver of this phenomenon.

Furthermore, the valuation of future prospects appears to be order-dependent, Observation #7: It makes a difference whether a risky future payoff is first devalued for risk and then for delay or in the opposite order (Öncüler and Onay 2009). When payoffs are discounted for risk first, they are assigned a less favorable value than in the reverse case. Moreover, the delay-first value practically coincides with the value reported when both dimensions are accounted for in one single operation. This finding #7 can be also interpreted as a manifestation of risk dependence of discounting.

5.2. Other Related Literature

There is a large empirical and theoretical literature on the domain of risk taking and an equally large one on time discounting, focusing on single aspects such as, for example, hyperbolic discounting, preferences for resolution timing, and the value of information. There are, in comparison, relatively few papers dealing with an integrated view of risk and time. However, the subject has recently gained traction. As reviewing this literature is beyond the scope of this paper, we focus on those contributions that are more closely related to our work.

Motivated by the similarities of anomalies in risk taking and time discounting behaviors, Prelec and Loewenstein (1991) develop psychological properties of multi-attribute prospect valuation that may be common in both decision domains. Thus, common ratio violations and decreasing impatience may be driven by the same psychological principles. The authors do not address how features of risk preferences and time preferences interact with each other, however.

Similarly, Quiggin and Horowitz (1995) analyze parallels between the theories of choice under risk and choice over time and show the usefulness of RDU for understanding the analogy between risk aversion and impatience. Leland and Schneider (2017) propose a different theory that can account for many anomalies in risk taking and time discounting behavior. Their approach extends the concept of salience from outcome differences to differences in probabilities and differences in delays. This enables the authors to explain a large set of interesting facts in risk taking, time discounting, and consumer behavior. However, they explicitly mention on page 20 that their theory "does not account for interaction effects between risk and time" that are precisely the object of our paper. On the other hand, our paper does not explain facts such as labeling effects, framing effects, or peanut effects, which are the explicanda of Leland and Schneider (2017)'s paper.

DeJarnette et al. (2020) study a setting that is complementary to ours: Their *time lotteries* have fixed prices, but random payment dates. In contrast, we explicitly abstract from uncertainty with regard to the timing of outcomes. However, extending our approach to their time-lottery setting may be an interesting direction of future research.

6. Concluding Remarks

We have demonstrated that our modeling approach organizes all seven phenomena of experimental research and is also able to accommodate a wide range of outcomes by plausible levels of survival probability. In our view, apart from explaining the seven regularities uncovered by experiments, the model helps to better understand the patterns of heterogeneity in individual behaviors. Not everyone is prone to common ratio violations. In fact, almost any kind of shape of probability weighting can be found in individual estimates, and even among common ratio violators the degree of subproportionality may vary greatly. So far, only a few contributions have already addressed the issue of heterogeneity (Epper, Fehr-Duda, and Bruhin 2011; Islam, Diecidue, and Hardardottir 2022). Thus, our framework provides a host of predictions that can be investigated in future experimental research. For example, people with comparatively stronger subproportional probability weights should, *ceteris paribus*, exhibit a greater increase in risk tolerance for delayed prospects than less subproportional decision makers do. Similarly, the former group should show a greater preference for uncertainty to resolve in the future rather than in the present. Moreover, these effects are predicted to be more pronounced for positively skewed prospects—a prediction that is specific to our model. Sequential resolution of uncertainty is another area where more work needs to be done as evidence on substantially delayed prospects is still missing. Ideally, the same subjects should be exposed to the full program of experiments delineated in this paper to find out if and when our predictions materialize.

Another interesting test of the model can be based on the model's assumption that survival probability depends on time horizon according to s^t. For a given participant sample at a given point in time, the preference parameters and the uncertainty perception s should not vary across time horizons. Thus, if a given participant sample faces future prospects of different delays, we should not observe a change in the estimated value of s (nor a change in estimated preference parameters) because such a change would challenge the assumption that survival probability can be represented by s^t.

The ultimate test of our model, however, is to exogenously manipulate the subjective probability that something may go wrong, s, the second crucial component of our approach aside from subproportionality. As effect sizes also depend on the perceived uncertainty of the future, such a manipulation can shed light on the question whether our model has actually identified an important causal driver of behavior.

Aside from conducting new experiments, the usefulness of our approach should be tested in the field as well. Both financial and insurance markets are fruitful areas for such an endeavor. Barberis (2013) concludes his review of 30 years of prospect theory in the following way: “Probability weighting, [...] has drawn increasing interest in recent years. Indeed, within the risk-related areas of finance, insurance, and gambling, probability weighting plays a more central role than loss aversion and has attracted significantly more empirical support” (p. 191). Thus, our survival-risk augmented version of probability weighting could be put to the test in these fields as well. Puzzles like the maturity dependence of risk premia may appear in a new light. Another fertile application may be option prices: Polkovnichenko and Zhao (2013) show the usefulness of probability weighting for explaining option prices, which could be enhanced by incorporating the maturity dimension as well.[19] Insurance markets are another domain where our approach may reconcile conflicting findings: Recognizing that risk preferences are delay-dependent may help understand why people are willing to pay outrageous premiums for certain insurance contracts, such as extended warranties, and totally unwilling to take out insurance at all, such as in the health domain.

We do not claim that subproportionality plus future uncertainty are the only important drivers in the domain of risk- and time-dependent decision making. Other factors such as concave utility, intrinsically hyperbolic pure time preferences, or reference dependence are also likely to play a role. However, there is accumulating evidence that risk and time preferences are intertwined and interact in systematic ways, and we are just beginning to understand the factors underlying these phenomena. We have shown that subproportionality plus subjectively perceived future uncertainty provides a unifying explanation for a set of key findings—suggesting that these factors should be taken seriously in future research.

References

Abdellaoui, Mohammed, A. Baillon, L. Placido, and Peter Wakker (2011a). “The Rich Domain of Uncertainty: Source Functions and Their Experimental Implementation.” *American Economic Review*, 101(2), 695–723.

Abdellaoui, Mohammed, Enrico Diecidue, and Ayse Öncüler (2011b). “Risk Preferences at Different Time Periods: An Experimental Investigation.” *Management Science*, 57, 975–987.

Abdellaoui, Mohammed, Emmanuel Kemel, Amma Panin, and Ferdinand M. Vieider (2018). “Take Your Time or Take Your Chance: Time Discounting as a Distorted Probability.” Working paper, Université de Louvain.

Abdellaoui, Mohammed, Peter Klibanoff, and Laetitia Placido (2015). “Experiments on Compound Risk in Relation to Simple Risk and to Ambiguity.” *Management Science*, 61, 1306–1322.

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Ahlbrecht, Martin and Martin Weber (1996). “The Resolution of Uncertainty: An Experimental Study.” *Journal of Institutional and Theoretical Economics*, 152, 593–607.
Ahlbrecht, Martin and Martin Weber (1997). “An Empirical Study on Intertemporal Decision Making under Risk.” *Management Science*, 43, 813–826.
Ainslie, George (1991). “Derivation of ’Rational’ Economic Behavior from Hyperbolic Discount Curves.” *American Economic Review, Papers and Proceedings*, 81(2), 334–340.
Allais, Maurice (1953). “Le Comportement de L’Homme Rationnel Devant le Risque: Critique des Postulats et Axiomes de L’Ecole Americaine.” *Econometrica*, 21, 503–546.
Arai, Dariush (1997). “Temporal Resolution of Uncertainty in Risky Choices.” *Acta Psychologica*, 96, 15–26.
Aydogan, Ilke, Han Bleichrodt, and Yu Gao (2016). “An Experimental Test of Reduction Invariance.” *Journal of Mathematical Psychology*, 75, 170–182.
Baillon, Aurelien, Olivier l’Haridon, and Laetitia Placido (2011). “Ambiguity Models and the Machina Paradoxes.” *American Economic Review*, 101(4), 1547–1560.
Barberis, Nicholas (2013). “Thirty Years of Prospect Theory in Economics: A Review and Assessment.” *Journal of Economic Perspectives*, 27(1), 173–196.
Baucells, Manel and Franz H. Heukamp (2010). “Common Ratio Using Delay.” *Theory and Decision*, 68, 149–158.
Baucells, Manel and Franz H. Heukamp (2012). “Probability and Time Tradeoff.” *Management Science*, 58, 831–842.
Bellemare, Charles, Michaela Krause, Sabine Kröger, and Chendi Zhang (2005). “Myopic Loss Aversion: Information Feedback vs. Investment Flexibility.” *Economics Letters*, 87, 319–324.
Benartzi, Shlomo and Richard Thaler (1995). “Myopic Loss Aversion and the Equity Premium Puzzle.” *Quarterly Journal of Economics*, 110, 73–92.
Benzion, Uri, Amnon Rapoport, and Joseph Yagil (1989). “Discount Rates Inferred from Decisions: An Experimental Study.” *Management Science*, 35, 270–284.
Blavatskyy, Pavlo and Ganna Pogrebna (2010). “Reevaluating Evidence on Myopic Loss Aversion: Aggregate Patterns versus Individual Choices.” *Theory and Decision*, 68, 159–171.
Bleichrodt, Han, K. I. M. Rohde, and Peter Wakker (2009). “Non-Hyperbolic Time Inconsistency.” *Games and Economic Behavior*, 66, 27–38.
Budescu, David and Ilan Fischer (2001). “The Same But Different: An Empirical Investigation of the Reducibility Principle.” *Journal of Behavioral Decision Making*, 14, 187–206.
Caplin, Andrew and John Leahy (2001). “Psychological Expected Utility Theory and Anticipatory Feelings.” *Quarterly Journal of Economics*, 116, 55–79.
Chakraborty, Anujit (2021). “Present Bias.” *Econometrica*, 89, 1921–1961.
Chakraborty, Anujit, Yoram Halevy, and Kota Saito (2020). “The Relation between Behavior under Risk and over Time.” *AER: Insights*, 2, 1–16.
Chew, Soo Hong and Larry G. Epstein (1989). “The Structure of Preferences and Attitudes towards the Timing of the Resolution of Uncertainty.” *International Economic Review*, 30, 103–117.
Chew, Soo Hong and Johanna L. Ho (1994). “Hope: An Empirical Study of Attitude Toward the Timing of Uncertainty Resolution.” *Journal of Risk and Uncertainty*, 8, 267–288.
Chung, Ngar-Kok, Detlof von Winterfeldt, and R. Duncan Luce (1994). “An Experimental Test of Event Commutativity in Decision Making under Uncertainty.” *Psychological Science*, 5, 394–400.
Coble, Keith H. and Jayson L. Lusk (2010). “At the Nexus of Risk and Time Preferences: An Experimental Investigation.” *Journal of Risk and Uncertainty*, 41, 67–79.
DeJarnette, Patrick, David Dillenberger, Daniel Gottlieb, and Pietro Ortoleva (2020). “Time Lotteries and Stochastic Impatience.” *Econometrica*, 88, 619–656.
Diecidue, Enrico and Peter P. Wakker (2001). “On the Intuition of Rank-Dependent Utility.” *Journal of Risk and Uncertainty*, 23, 281–298.
Dillenberger, David (2010). “Preferences for One-Shot Resolution of Uncertainty.” *Econometrica*, 78, 1973–2004.

Dillenberger, David and Uzi Segal (2014). “Recursive Ambiguity and Machina’s Examples.” *International Economic Review*, 56, 55–61.

Dohmen, Thomas, Armin Falk, David Huffman, and Uwe Sunde (2017). “The Robustness and Pervasiveness of Sub-Additivity in Intertemporal Choice.” Working Paper, Briq Institute on Behavior & Inequality.

Ebert, Jane E. J. and Drazen Prelec (2007). “The Fragility of Time: Time-Insensitivity and Valuation of the Near and Far Future.” *Management Science*, 53, 1423–1438.

Eisenbach, Thomas M. and Martin C. Schmalz (2016). “Anxiety in the Face of Risk.” *Journal of Financial Economics*, 121, 414–426.

Eliaz, Kfir and Andrew Schotter (2007). “Experimental Testing of Intrinsic Preferences for Noninstrumental Information.” *American Economic Review, Papers and Proceedings*, 97(2), 166–169.

Epper, Thomas F and Helga Fehr-Duda (2012). “The Missing Link: Unifying Risk Taking and Time Discounting.” University of Zurich, Department of Economics, Working paper No. 96.

Epper, Thomas F., Helga Fehr-Duda, and Adrian Bruhin (2009). “Uncertainty Breeds Decreasing Impatience: The Role of Risk Preferences in Time Discounting.” Institute for Empirical Research in Economics, University of Zurich, Working paper No. 412.

Epper, Thomas F., Helga Fehr-Duda, and Adrian Bruhin (2011). “Viewing the Future through a Warped Lens: Why Uncertainty Generates Hyperbolic Discounting.” *Journal of Risk and Uncertainty*, 43, 169–203.

Epstein, Larry and Stanley Zin (1991). “Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: An Empirical Analysis.” *Journal of Political Economy*, 99, 263–286.

Epstein, Larry G. (2008). “Living with Risk.” *Review of Economic Studies*, 75, 1121–1141.

Epstein, Larry G. and Igor Kopylov (2007). “Cold Feet.” *Theoretical Economics*, 2, 231–259.

Ericson, Keith M. Marzilli and Laibson David (2019). “Intertemporal Choice.” *Bernheim, D., Laibson, D., DellaVigna, S.: Handbook of Behavioral Economics —Foundations and Applications 1*, Elsevier, 2, 1–67.

Fan, Yuyu, David V. Budescu, and Enrico Diecidue (2019). “Decisions With Compound Lotteries.” *Decision*, 6, 109–133.

Fehr-Duda, Helga and Thomas F. Epper (2012). “Probability and Risk: Foundations and Economic Implications of Probability-Dependent Risk Preferences.” *Annual Review of Economics*, 4, 567–593.

Frederick, Shane, George F. Loewenstein, and Ted O’Donoghue (2002). “Time Discounting and Time Preference: A Critical Review.” *Journal of Economic Literature*, 40, 351–401.

Gabaix, Xavie and David Laibson (2022). “Myopia and Discounting.” NBER Working Paper 23254.

Ganguly, Ananda and Joshua Tasoff (2017). “Fantasy and Dread: The Demand for Information and the Consumption Utility of the Future.” *Management Science*, 63, 4037–4060.

Gneezy, Uri, Arie Kapteyn, and Jan Potters (2003). “Evaluation Periods and Asset Prices in a Market Experiment.” *Journal of Finance*, LVIII, 821–837.

Gneezy, Uri and Jan Potters (1997). “An Experiment on Risk Taking and Evaluation Periods.” *Quarterly Journal of Economics*, 112, 631–645.

Gonzalez, Richand and George Wu (1999). “On the Shape of the Probability Weighting Function.” *Cognitive Psychology*, 38, 129–166.

Grant, Simon, Atshushi Kajii, and Ben Polak (2000). “Temporal Resolution of Uncertainty and Recursive Non-expected Utility Models.” *Econometrica*, 68, 425–434.

Gul, Faruk (1991). “A Theory of Disappointment Aversion.” *Econometrica*, 59, 667–686.

Haigh, Michael and John List (2005). “Do Professional Traders Exhibit Myopic Loss Aversion? An Experimental Analysis.” *Journal of Finance*, 60, 523–534.

Halevy, Yoram (2008). “Strotz Meets Allais: Diminishing Impatience and Certainty Effect.” *American Economic Review*, 98(3), 1145–1162.

Halevy, Yoram (2015). “Time Consistency: Stationarity and Time Invariance.” *Econometrica*, 83, 335–352.

Islam, Marco, Enrico Diecidue, and Hjördis Hardardottir (2022). *Why Do People Discount? The Role of Impatience and Future Uncertainty*, chap. Part IV, Doctoral Dissertation of Marco Islam. Lund University.
Jones, Edward E. and C. Anderson Johnson (1973). "Delay of Consequences and the Riskiness of Decisions." *Journal of Personality*, 41, 613–637.
Kahneman, Daniel and Amos Tversky (1979). "Prospect Theory: An Analysis of Decision under Risk." *Econometrica*, 47, 263–292.
Keren, Gideo and Peter Roelofsma (1995). "Immediacy and Certainty in Intertemporal Choice." *Organizational Behavior and Human Decision Processes*, 63, 287–297.
Kőszegi, Botond and Adam Szeidl (2013). "A Model of Focusing in Economic Choice." *The Quarterly Journal of Economics*, 128, 53–104.
Kreps, David M. and Evan L. Porteus (1978). "Temporal Resolution of Uncertainty and Dynamic Choice Theory." *Econometrica*, 46, 185–200.
Laibson, David (1997). "Golden Eggs and Hyperbolic Discounting." *Quarterly Journal of Economics*, 112, 443–477.
Langer, Thomas and Martin Weber (2005). "Myopic Prospect Theory vs. Myopic Loss Aversion: How General is the Phenomenon?" *Journal of Economic Behavior and Organization*, 56, 25–38.
Leland, Jonathan and Mark Schneider (2017). "Risk Preference, Time Preference, and Salience Perception." ESI Working Paper 17–16, Chapman University.
Loewenstein, George F. and Richard Thaler (1989). "Anomalies: Intertemporal Choice." *Journal of Economic Perspectives*, 3(4), 181–193.
Lovallo, Dan and Daniel Kahneman (2000). "Living with Uncertainty: Attractiveness and Resolution Timing." *Journal of Behavioral Decision Making*, 13, 179–190.
Machina, Mark (2009). "Risk, Ambiguity, and the Rank-Dependence Axioms." *American Economic Review*, 99(1), 385–392.
Machina, Mark (2014). "Ambiguity Aversion with Three or More Outcomes." *American Economic Review*, 104(12), 3814–3840.
Nielsen, Kirby (2020). "Preferences for the Resolution of Uncertainty and the Timing of Information." *Journal of Economic Theory*, 189, 1–39.
Noussair, Charles and Ami Ping Wu (2006). "Risk Tolerance in the Present and the Future: An Experimental Study." *Managerial and Decision Economics*, 27, 401–412.
O'Donoghue, Ted and Jason Somerville (2018). "Modeling Risk Aversion in Economics." *Journal of Economic Perspectives*, 32(2), 91–114.
Öncüler, Ayse and Selçuk Onay (2009). "How Do We Evaluate Future Gambles? Experimental Evidence on Path Dependency in Risky Intertemporal Choice." *Journal of Behavioral Decision Making*, 22, 280–300.
Polkovnichenko, Valery and Feng Zhao (2013). "Probability Weighting Functions Implied in Options Prices." *Journal of Financial Economics*, 107, 580–609.
Prelec, Drazen (1998). "The Probability Weighting Function." *Econometrica*, 66, 497–527.
Prelec, Drazen and George F. Loewenstein (1991). "Decision Making over Time and under Uncertainty: A Common Approach." *Management Science*, 37, 770–786.
Quiggin, John (1982). "A Theory of Anticipated Utility." *Journal of Economic Behavior and Organization*, 3, 323–343.
Quiggin, John and John Horowitz (1995). "Time and Risk." *Journal of Risk and Uncertainty*, 10, 37–55.
Rachlin, Howard, Anders Raineri, and David Cross (1991). "Subjective Probability and Delay." *Journal of the Experimental Analysis of Behavior*, 55, 233–244.
Read, Daniel (2001). "Is Time-Discounting Hyperbolic or Subadditive?" *Journal of Risk and Uncertainty*, 23, 5–32.
Read, Daniel and Peter H. M. P. Roelofsma (2003). "Subadditive versus Hyperbolic Discounting: A Comparison of Choice and Matching." *Organizational Behavior and Human Decision Processes*, 91, 140–153.

Sagristano, Michael D., Yaacov Trope, and Nira Liberman (2002). "Time-Dependent Gambling: Odds Now, Money Later." *Journal of Experimental Psychology*, 131, 364–376.
Saito, Kota (2011). "Strotz Meets Allais: Diminishing Impatience and the Certainty Effect: Comment." *American Economic Review*, 101(5), 2271–2275.
Segal, Uzi (1987a). "Some Remarks on Quiggin's Anticipated Utility." *Journal of Economic Behavior and Organization*, 8, 145–154.
Segal, Uzi (1987b). "The Ellsberg Paradox and Risk Aversion: An Anticipated Utility Approach." *International Economic Review*, 28, 175–202.
Segal, Uzi (1990). "Two-Stage Lotteries Without the Reduction Axiom." *Econometrica*, 58, 349–377.
Shelley, Marjorie K. (1994). "Gain/Loss Asymmetry in Risky Intertemporal Choice." *Organizational Behavior and Human Decision Processes*, 59, 124–159.
Starmer, Chris (2000). "Developments in Non-Expected Utility Theory: The Hunt for a Descriptive Theory of Choice under Risk." *Journal of Economic Literature*, 38, 332–382.
Stevenson, Mary Kay (1992). "The Impact of Temporal Context and Risk on the Judged Value of Future Outcomes." *Organizational Behavior and Human Decision Processes*, 52, 455–491.
Thaler, Richard H., Amos Tversky, Daniel Kahneman, and Alan Schwartz (1997). "The Effect of Myopia and Loss Aversion on Risk Taking: An Experimental Test." *Quarterly Journal of Economics*, 112, 647–661.
Tversky, Amos and Daniel Kahneman (1992). "Advances in Prospect Theory: Cumulative Representation of Uncertainty." *Journal of Risk and Uncertainty*, 5, 297–323.
van Binsbergen, Jules, Michael Brandt, and Ralph Koijen (2012). "On the Timing and Pricing of Dividends." *American Economic Review*, 102(4), 1596–1618.
von Gaudecker, Hans-Martin, Arthur van Soest, and Erik Wengström (2011). "Heterogeneity in Risky Choice Behaviour in a Broad Population." *American Economic Review*, 101(2), 664–694.
Wakker, Peter (1988). "Nonexpected Utility as Aversion of Information." *Journal of Behavioral Decision Making*, 1, 169–175.
Wakker, Peter (2010). *Prospect Theory for Risk and Ambiguity*. Cambridge University Press, Cambridge.
Walther, Herbert (2003). "Normal-Randomness Expected Utility, Time Preference and Emotional Distortions." *Journal of Economic Behavior and Organization*, 52, 253–266.
Weber, Bethany J. and Gretchen B. Chapman (2005). "The Combined Effects of Risk and Time on Choice: Does Uncertainty Eliminate the Immediacy Effect? Does Delay Eliminate the Certainty Effect?" *Organizational Behavior and Human Decision Processes*, 96, 104–118.
Wu, George (1999). "Anxiety and Decision Making with Delayed Resolution of Uncertainty." *Theory and Decision*, 46, 159–198.

Supplementary Material

Supplementary data are available at *JEEA* online.

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