Electronic Supplementary Material

For

The Uncertainty Triangle – Uncovering Heterogeneity in Attitudes Towards Uncertainty

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A Analytical Details

A.1 Testing For GARP

Varian (1982) provides a convenient algorithm for testing the Generalized Axiom of Revealed Preferences (GARP) and, in general, we adopt his notation and methods here. Following Burghart (2020), testing GARP in this setting is computationally less intensive, and more intuitive, if we interpret lower envelope lotteries as two-dimensional demand vectors, relative to the worst outcome. To do this we transform lower envelope lotteries into two dimensions. Denoting the lower envelope lottery selected from the i^{th} budget as L^{i*}, we use the following transformation:

$$L^{i*} = \left(\underline{p}_{60}^{i*}, \underline{p}_{20}^{i*}, y^{i*}\right) \rightarrow \left(\underline{p}_{60}^{i*} + y^{i*}, \underline{p}_{60}^{i*}\right) = \left(x_1^{i*}, x_2^{i*}\right) = \mathbf{x}^{i*} \tag{15}$$

This transformation means that $\mathbf{x}^{i*}$ can be interpreted as a two-dimensional demand vector, relative to the worst outcome. Denote as $\mathbf{q}^i$ the two-dimensional vector of prices that describes the linear budget from which $\mathbf{x}^{i*}$ was selected.

The starting point for testing GARP is to construct a directly revealed preferred graph. For the 25 budgets in our experiment, and each participants choices $\left(\mathbf{x}^{1*}, \ldots, \mathbf{x}^{25*}\right)$, we construct a 25 by 25 matrix M (the directly revealed graph) whose ij^{th} entry is given by

$$m_{ij} = \begin{cases} 1 & \text{if } \mathbf{q}^i \cdot \mathbf{x}^{i*} \geq \mathbf{q}^i \cdot \mathbf{x}^{j*} \\ 0 & \text{otherwise} \end{cases} \tag{16}$$

We then construct the indirectly revealed preferred graph MT, which is the transitive closure of M. The ij^{th} entry of the closure is given by

$$mt_{ij} = \begin{cases} 1 & \text{if } m_{ij}^{25} > 0 \\ 0 & \text{otherwise} \end{cases} \tag{17}$$

where m_{ij}^{25} is the ij^{th} entry of the matrix $M^{25} = MM \cdots M$. If $mt_{ij} = 1$ and $\mathbf{q}^j \cdot \mathbf{x}^{j*} > \mathbf{q}^j \cdot \mathbf{x}^{i*}$ for some i and j (with $\mathbf{x}^{i*} \neq \mathbf{x}^{j*}$) there is a GARP violation.

To calculate Houtman-Maks, conditional on observing at least one violation of GARP, we use a brute force approach. We check whether all subsets of size 24 are consistent with GARP (using the above algorithm). If no subset of size 24 is GARP compliant we take all subsets of size 23 and check whether any of these subsets are GARP compliant. We proceed in such a manner until we find at least one subset of the data that is GARP compliant. The cardinality of that subset is the Houtman-Maks.

A.2 Testing For Partial Ignorance Expected Utility

The Partial Ignorance Expected Utility model gives rise to indifference curves that linear and parallel in the uncertainty triangle. So, given a choice from one budget, the PEU indifference curve structure lets us make predictions about choices from other budgets. To make predictions, we need to compare the steepness of the i^{th} budget, denoted by the ratio of its prices $\left(\frac{q_1}{q_2}\right)^i$, to the steepness of the j^{th} budget, denoted by the ratio of its prices $\left(\frac{q_1}{q_2}\right)^j$. We say:

1. The j^{th} budget is flatter than the i^{th} budget whenever $\left(\frac{q_1}{q_2}\right)^j < \left(\frac{q_1}{q_2}\right)^i$
2. The j^{th} budget is steeper than the i^{th} budget whenever $\left(\frac{q_1}{q_2}\right)^j > \left(\frac{q_1}{q_2}\right)^i$

We can use these steepness comparisons to make an exhaustive list of predictions. Using $\mathbf{x}^{i*}$ to denote the choice from the i^{th} budget:

- **Case 1:** The choice from the i^{th} budget is the most uncertain alternative available (i.e. the alternative was on the horizontal or vertical leg of the uncertainty triangle). Denote this "corner" alternative by $\lfloor \mathbf{x} \rfloor^i$

$$\mathbf{x}^{i*} = \lfloor \mathbf{x} \rfloor^i \;\rightarrow\; \begin{cases} \mathbf{x}^j = \lfloor \mathbf{x} \rfloor^j & \text{when } \left(\frac{q_1}{q_2}\right)^j < \left(\frac{q_1}{q_2}\right)^i \\ \text{no prediction} & \text{when } \left(\frac{q_1}{q_2}\right)^j > \left(\frac{q_1}{q_2}\right)^i \\ \text{no prediction} & \text{when } \left(\frac{q_1}{q_2}\right)^j = \left(\frac{q_1}{q_2}\right)^i \end{cases}$$

- **Case 2:** The choice from i^{th} budget fully-specified lottery (i.e. the alternative selected was on the hypotenuse in the uncertainty triangle). Denote this "corner" alternative by $\lceil \mathbf{x} \rceil^i$

$$\mathbf{x}^{i*} = \lceil \mathbf{x} \rceil^i \;\rightarrow\; \begin{cases} \text{no prediction} & \text{when } \left(\frac{q_1}{q_2}\right)^j < \left(\frac{q_1}{q_2}\right)^i \\ \mathbf{x}^j = \lceil \mathbf{x} \rceil^j & \text{when } \left(\frac{q_1}{q_2}\right)^j > \left(\frac{q_1}{q_2}\right)^i \\ \text{no prediction} & \text{when } \left(\frac{q_1}{q_2}\right)^j = \left(\frac{q_1}{q_2}\right)^i \end{cases}$$

- **Case 3:** The choice from the i^{th} budget is on the "interior" (i.e. not Case 1 or Case 2). Denote this set of alternatives by $[\mathbf{x}]^i$

$$\mathbf{x}^{i*} \in [\mathbf{x}]^i \;\rightarrow\; \begin{cases} \mathbf{x}^j = \lfloor \mathbf{x} \rfloor^j & \text{when } \left(\frac{q_1}{q_2}\right)^j < \left(\frac{q_1}{q_2}\right)^i \\ \mathbf{x}^j = \lceil \mathbf{x} \rceil^j & \text{when } \left(\frac{q_1}{q_2}\right)^j > \left(\frac{q_1}{q_2}\right)^i \\ \text{no prediction} & \text{when } \left(\frac{q_1}{q_2}\right)^j = \left(\frac{q_1}{q_2}\right)^i \end{cases}$$

This provides an exhaustive list of predictions under an PEU representation.

Given a set of predictions, it seems reasonable to assess two conditions:

- **Condition 1:** Predictions about choices are internally consistent.
- **Condition 2:** Choices are consistent with the predictions.

The first condition requires that choices from distinct budgets do not generate conflicting predictions about the choice from a third budget (also distinct). The second condition simply requires that choices are consistent with the set of (internally consistent) predictions. We say that choices are consistent with an PEU representation if these two conditions are met.

Algorithmically, we test the Condition 1 by constructing three square matrices, $\lfloor D \rfloor$, $\lceil D \rceil$, and $[D]$, with entries defined, respectively:

$$\lfloor d \rfloor_{ij} = \begin{cases} -1 & \text{if } \mathbf{x}^{i*} = \lfloor \mathbf{x} \rfloor^i \text{ and } \left(\frac{q_1}{q_2}\right)^j < \left(\frac{q_1}{q_2}\right)^i \\ 0 & \text{otherwise} \end{cases} \tag{18}$$

$$\lceil d \rceil ij = \begin{cases} +1 & \text{if } \mathbf{x}^{i*} = \lceil \mathbf{x} \rceil^i \text{ and } \left(\frac{q_1}{q_2}\right)^j > \left(\frac{q_1}{q_2}\right)^i \\ 0 & \text{otherwise} \end{cases} \tag{19}$$

$$[d]_{ij} = \begin{cases} -1 & \text{if } \mathbf{x}^{i*} \in [\mathbf{x}]^i \text{ and } \left(\frac{q_1}{q_2}\right)^j < \left(\frac{q_1}{q_2}\right)^i \\ +1 & \text{if } \mathbf{x}^{i*} \in [\mathbf{x}]^i \text{ and } \left(\frac{q_1}{q_2}\right)^j > \left(\frac{q_1}{q_2}\right)^i \\ 0 & \text{otherwise} \end{cases} \tag{20}$$

Essentially, these three matrices document whether a choice from the j^{th} budget is predicted to be the most uncertain alternative available (-1), the fully-specified lottery $(+1)$, or neither (0), based upon some original choice ($\mathbf{x}^i$) and steepness of that original budget $\left(\frac{q_1}{q_2}\right)^i$.

Because $[D]$ is the only matrix that has both $+1$ and -1 entries (i.e. only when $\mathbf{x}^{i*} \in [\mathbf{x}]^i$ can we get predictions of both $\lfloor \mathbf{x} \rfloor^j$ and $\lceil \mathbf{x} \rceil^j$) we first check it for internal consistency of its predictions. We construct two row vectors, Max$[D]$ and Min$[D]$, where

$$\begin{aligned} \text{Max}[D] &= [\text{Max}\{[d]_{\cdot 1}, 0\}, \text{Max}\{[d]_{\cdot 2}, 0\}, \ldots, \text{Max}\{[d]_{\cdot J}, 0\}] \\ \text{Min}[D] &= [\text{Min}\{[d]_{\cdot 1}, 0\}, \text{Min}\{[d]_{\cdot 2}, 0\}, \ldots, \text{Min}\{[d]_{\cdot J}, 0\}] \end{aligned} \tag{21}$$

where $[d]_{\cdot j}$ indicates the collection of entries in the j^{th} column of $[D]$. Notice that for the element-by-element multiplication, Max$[d]_j *$ Min$[d]_j \geq 0, \ \forall j = 1, \ldots, J$ if and only if predictions are internally consistent in $[D]$.

Conditional on $[D]$ exhibiting internal prediction consistency, all that remains is to verify that all three of the following conditions hold:

$$\begin{aligned} \text{Max}\lceil d \rceil_j \cdot \text{Min}\lfloor d \rfloor_j &\geq 0, \ \forall j = 1, \ldots, J \\ \text{Max}\lceil d \rceil_j \cdot \text{Min}[d]_j &\geq 0, \ \forall j = 1, \ldots, J \\ \text{Max}[d]_j \cdot \text{Min}\lfloor d \rfloor_j &\geq 0, \ \forall j = 1, \ldots, J \end{aligned} \tag{22}$$

where

$$\begin{aligned} \text{Max}\lceil D \rceil &= [\text{Max}\{\lceil d \rceil_{\cdot 1}\}, \text{Max}\{\lceil d \rceil_{\cdot 2}\}, \ldots, \text{Max}\{\lceil d \rceil_{\cdot J}\}] \\ \text{Min}\lfloor D \rfloor &= [\text{Min}\{\lfloor d \rfloor_{\cdot 1}\}, \text{Min}\{\lfloor d \rfloor_{\cdot 2}\}, \ldots, \text{Min}\{\lfloor d \rfloor_{\cdot J}\}] \end{aligned} \tag{23}$$

If predictions are internally consistent (i.e. choices satisfy condition 1) it is a straightforward exercise to verify that choices are consistent with predictions (i.e. choices satisfy condition 2). If choices satisfy both condition 1 and condition 2, they pass our test for a Partial Ignorance Expected Utility representation.

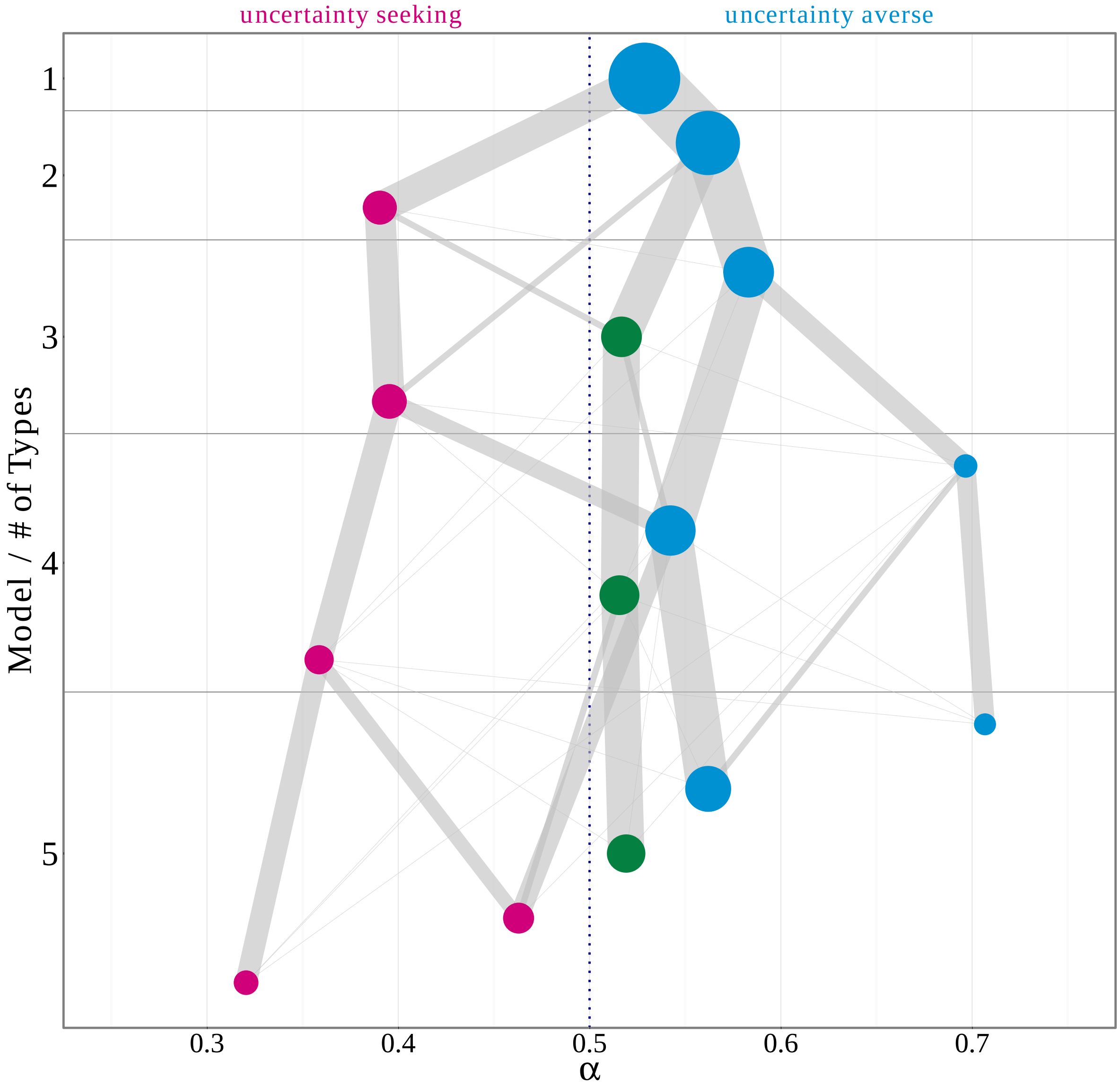
B Classification

B.1 Partial Ignorance Expected Utility Parameter Estimates

B.1.1 Partial Ignorance Expected Utility Transition Probabilities

The weighted graph in Figure 16 depicts transitions of participants between preference types when increasing the total number of types (C) in the finite mixture model. For a model with n types, the weight of the edges (i.e. the thickness of the lines) illustrates the fraction of participants moving to the $n+1$ types in the subsequent model below. The proportion of participants assigned to the uncertainty neutral (**green**) type is remarkably robust for $C \geq 3$. Relative to $C = 3$, more extreme preference types emerge for the uncertainty averse and uncertainty seeking types when $C = 4$ and $C = 5$. The overall proportions of participants in the uncertainty averse and uncertainty seeking categories is, however, remarkably consistent for $C \geq 3$.

Figure 16: Transition Probabilities For PEU Preference Types Assuming One to Five Types (i.e. $C = 1, \ldots, 5$)



B.1.2 Partial Ignorance Expected Utility Posterior Probabilities

Figures 17 and 18 depict histograms of the posterior probabilities for the two models presented in the main text. The posterior probability that an individual i choosing from the set of lower envelope lotteries $\mathbb{L}_i$ belongs to type c is defined as (see Section 3.2.1 for notational definitions):

$$\tau_{ic} = \frac{\pi_c f(\mathbb{L}_i; \alpha_c, \sigma_c)}{\sum_{c=1}^{C} \pi_c f(\mathbb{L}_i; \alpha_c, \sigma_c)} \,. \tag{24}$$

The histograms in Figures 17 and 18 provide a positive impression regarding how well our classification procedure works. Specifically, the vast majority of participants are clearly assigned to one type because posterior probabilities are near $\tau = 1$ or $\tau = 0$. This clear assignment to one type or another is an important validation for finite mixture methods – if assignment to types is unclear, such that posterior probabilities are away from the bounds, then using finite mixture methods would be inappropriate.

Figure 17: Posterior Probabilities for PEU

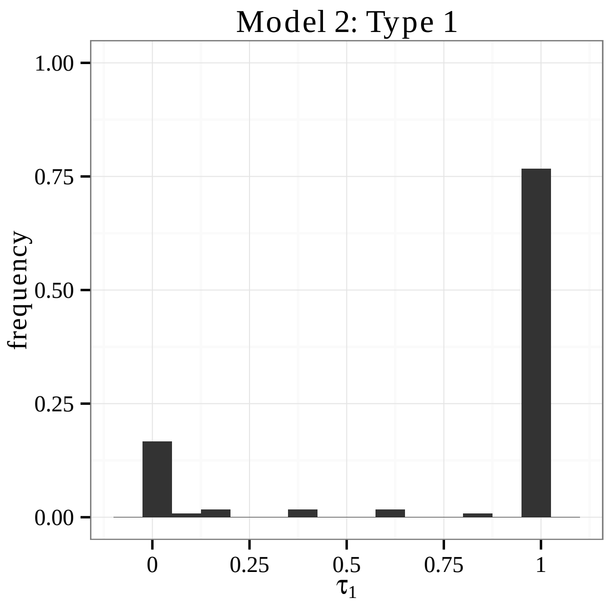
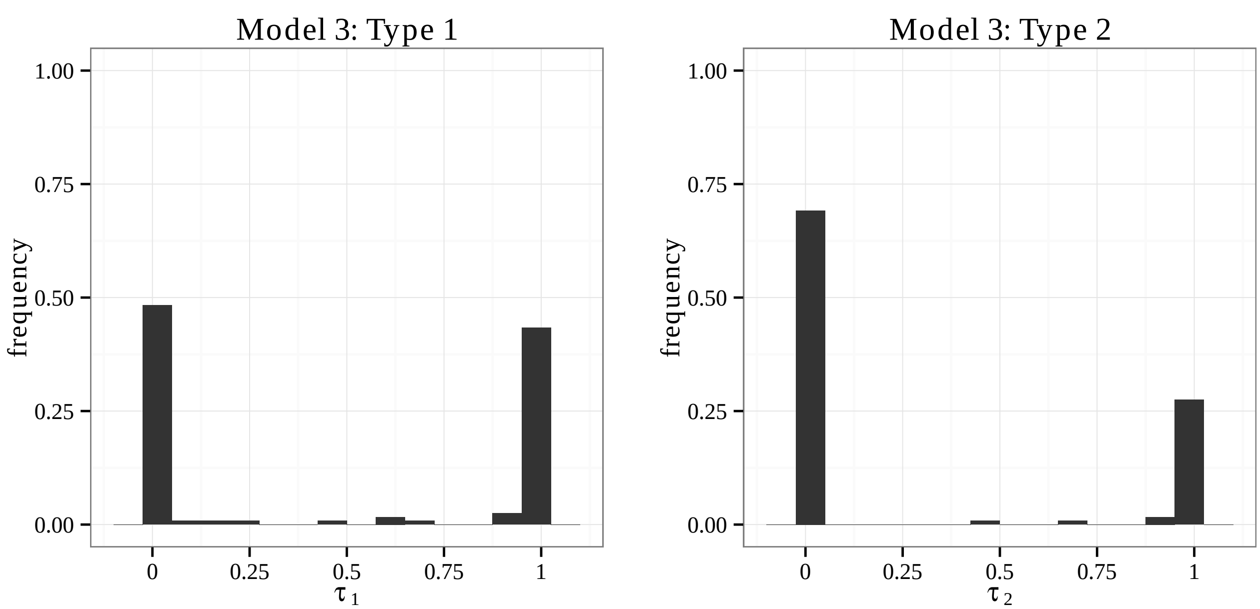


Figure 18: Posterior Probabilities for PEU



B.2 β-Partial Ignorance Expected Utility

B.2.1 β-Partial Ignorance Expected Utility Transition Probabilities

Figure 19 shows the transition probabilities when the type count is increased from one to two. The vertex of the graph labeled by 1 corresponds to the parameter estimate for the homogenous preference model. When allowing for two types, a fraction of subjects is allocated to the lower type (vertex labeled by 2 at the bottom right), whereas the remaining subjects are allocated to the upper type (vertex labeled by 2 on the $\alpha = 0.50$ line). The size of the vertices corresponds to the posterior probability of being assigned to one of the types. The thicker the edge linking the vertex labeled by 1 and 2, the higher the transition probability. When increasing from one type to two types there is a split of the homogeneous preferences (i.e. the ’red 1’) into two qualitatively distinct preference types (i.e. the ’red 2’ and the ’green 2’). Figure 20 shows the transition probabilities when the type count is increased from two to three. When increasing from two to three types the ’red 2’ type splits about equally into two ’red 3’ types. The proximal ’blue 3’ types are not qualitatively distinct from the ’green 2’ type.

Figure 19: Preference Types for β-PEU-types Assuming One to Two Types

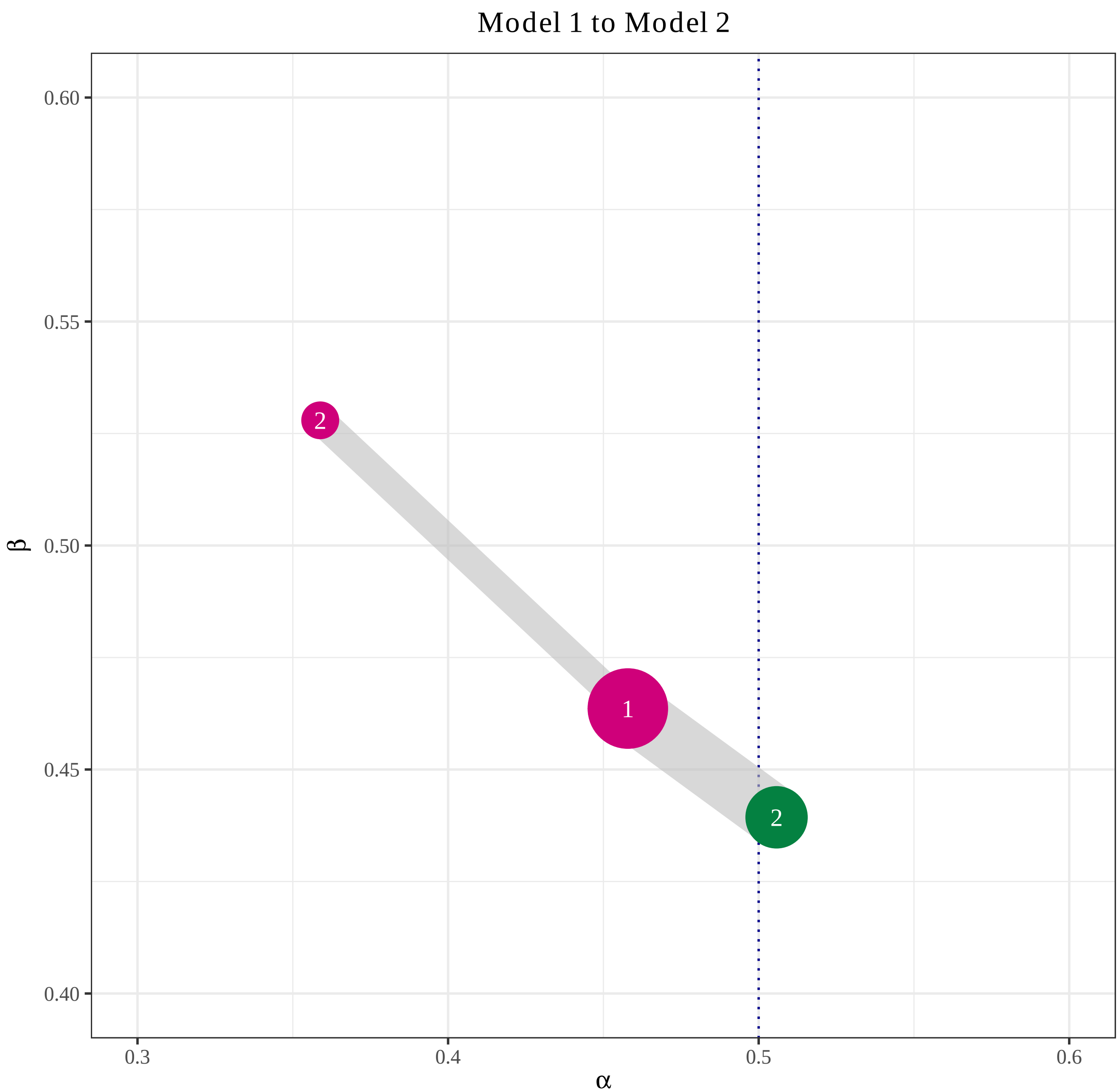
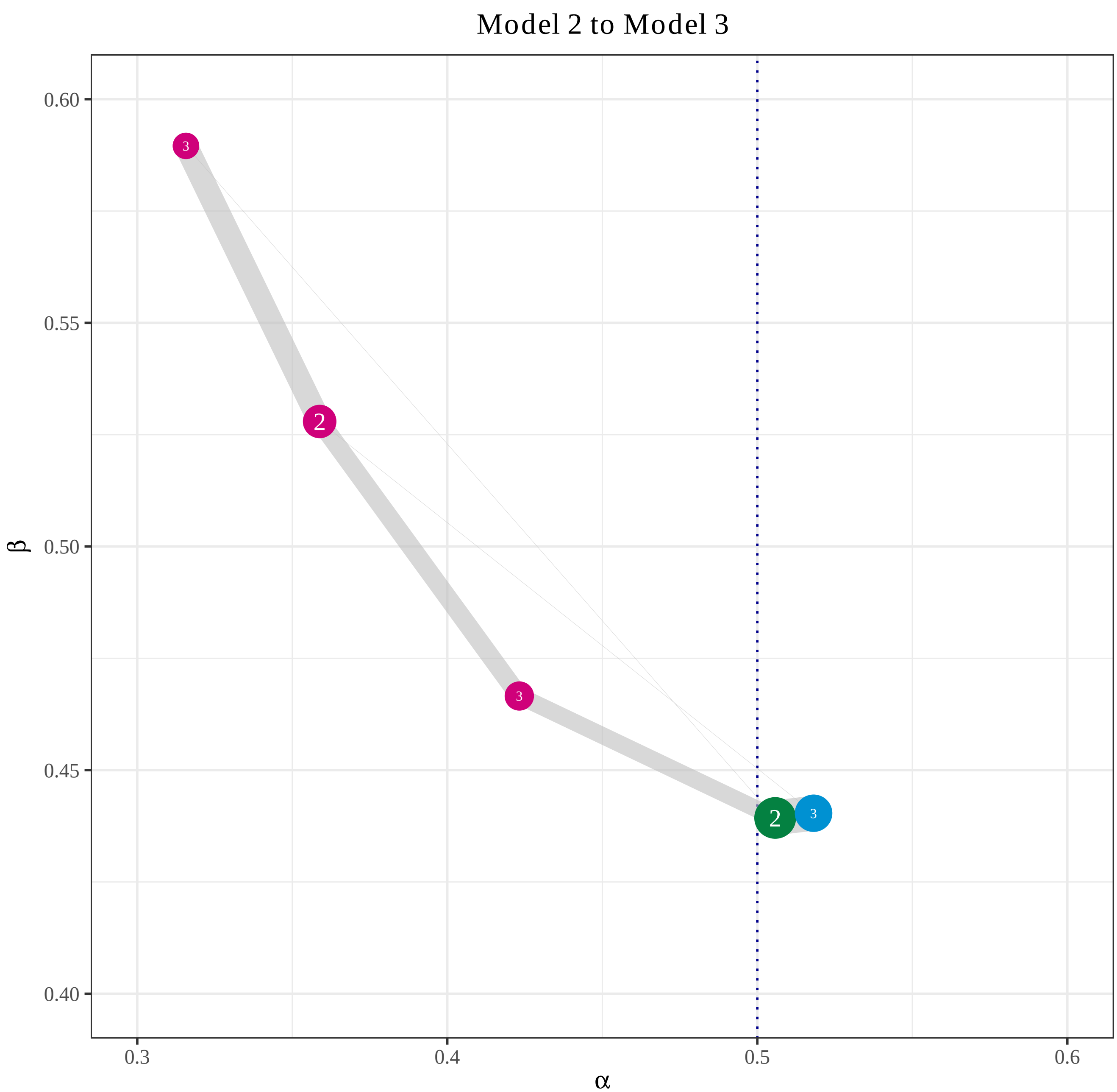


Figure 20: Preference Types for β-PEU-types Assuming Two to Three Types



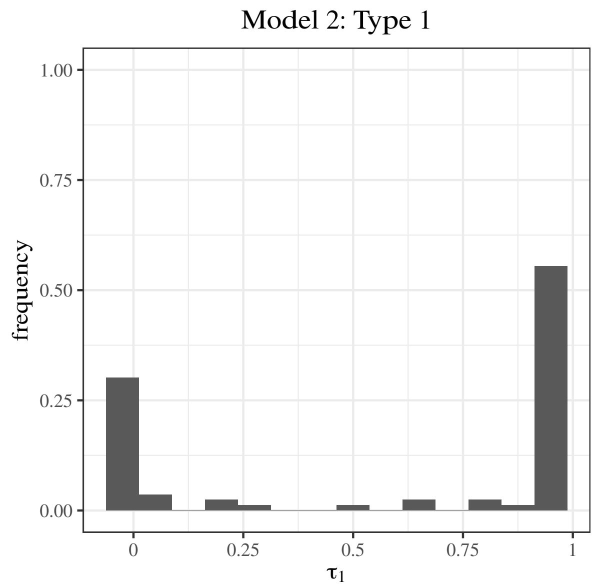
B.2.2 β-Partial Ignorance Expected Utility Posterior Probabilities

The posterior probability that an individual i choosing from the set of lower envelope lotteries $\mathbb{L}_i$ belongs to type c is defined as:

$$\tau_{ic} = \frac{\pi_c f(\mathbb{L}_i; \alpha_c, \beta_c, \sigma_c)}{\sum_{c=1}^{C} \pi_c f(\mathbb{L}_i; \alpha_c, \beta_c, \sigma_c)} \,. \tag{25}$$

The histograms in Figures 21 show how well our classification procedure works. As with the posteriors for PEU, we find that the majority of participants are clearly assigned to one type because posterior probabilities are near $\tau = 1$ or $\tau = 0$.

Figure 21: Posterior Probabilities for β-PEU



C Experimental Details

C.1 Choice Situation Details

Table 3 lists normalized prices q_1, q_2, their ratio $\frac{q_1}{q_2}$, and likelihood l for each budget.

Table 3: Choice Situation (CS) Budget Details

CS	q_1	q_2	l	$\frac{q_1}{q_2}$
1	0.2	0.8	0.2	0.25
2	0.3	0.7	0.3	0.43
3	0.4	0.6	0.4	0.67
4	0.5	0.5	0.5	1.00
5	0.6	0.4	0.6	1.50
6	0.7	0.3	0.7	2.33
7	0.8	0.2	0.8	4.00
8	0.2	0.6	0.2	0.33
9	0.3	0.5	0.3	0.60
10	0.4	0.4	0.4	1.00
11	0.5	0.3	0.5	1.67
12	0.6	0.2	0.6	3.00
13	0.2	0.6	0.4	0.33
14	0.3	0.5	0.5	0.60
15	0.4	0.4	0.6	1.00
16	0.5	0.3	0.7	1.67
17	0.6	0.2	0.8	3.00
18	0.2	0.4	0.2	0.50
19	0.3	0.3	0.3	1.00
20	0.4	0.2	0.4	2.00
21	0.5	0.1	0.5	5.00
22	0.1	0.5	0.5	0.20
23	0.2	0.4	0.6	0.50
24	0.3	0.3	0.7	1.00
25	0.4	0.2	0.8	2.00

C.2 Exhaustive List of Each Lower Envelope Lottery

Table 4: An Exhaustive List of the Lower Envelope Lotteries Available in Each Choice Situation

	Risky End					Uncertain End
CS	$\delta = 1.0$	$\delta = 0.8$	$\delta = 0.6$	$\delta = 0.4$	$\delta = 0.2$	$\delta = 0.0$
1	$(0.2, 0.8, 0)$	$(0.16, 0.64, 0.2)$	$(0.12, 0.48, 0.4)$	$(0.08, 0.32, 0.6)$	$(0.04, 0.16, 0.8)$	$(0, 0, 1)$
2	$(0.3, 0.7, 0)$	$(0.24, 0.56, 0.2)$	$(0.18, 0.42, 0.4)$	$(0.12, 0.28, 0.6)$	$(0.06, 0.14, 0.8)$	$(0, 0, 1)$
3	$(0.4, 0.6, 0)$	$(0.32, 0.48, 0.2)$	$(0.24, 0.36, 0.4)$	$(0.16, 0.24, 0.6)$	$(0.08, 0.12, 0.8)$	$(0, 0, 1)$
4	$(0.5, 0.5, 0)$	$(0.4, 0.4, 0.2)$	$(0.3, 0.3, 0.4)$	$(0.2, 0.2, 0.6)$	$(0.1, 0.1, 0.8)$	$(0, 0, 1)$
5	$(0.6, 0.4, 0)$	$(0.48, 0.32, 0.2)$	$(0.36, 0.24, 0.4)$	$(0.24, 0.16, 0.6)$	$(0.12, 0.08, 0.8)$	$(0, 0, 1)$
6	$(0.7, 0.3, 0)$	$(0.56, 0.24, 0.2)$	$(0.42, 0.18, 0.4)$	$(0.28, 0.12, 0.6)$	$(0.14, 0.06, 0.8)$	$(0, 0, 1)$
7	$(0.8, 0.2, 0)$	$(0.64, 0.16, 0.2)$	$(0.48, 0.12, 0.4)$	$(0.32, 0.08, 0.6)$	$(0.16, 0.04, 0.8)$	$(0, 0, 1)$
8	$(0.2, 0.8, 0)$	$(0.16, 0.68, 0.16)$	$(0.12, 0.56, 0.32)$	$(0.08, 0.44, 0.48)$	$(0.04, 0.32, 0.64)$	$(0, 0.2, 0.8)$
9	$(0.3, 0.7, 0)$	$(0.24, 0.6, 0.16)$	$(0.18, 0.5, 0.32)$	$(0.12, 0.4, 0.48)$	$(0.06, 0.3, 0.64)$	$(0, 0.2, 0.8)$
10	$(0.4, 0.6, 0)$	$(0.32, 0.52, 0.16)$	$(0.24, 0.44, 0.32)$	$(0.16, 0.36, 0.48)$	$(0.08, 0.28, 0.64)$	$(0, 0.2, 0.8)$
11	$(0.5, 0.5, 0)$	$(0.4, 0.44, 0.16)$	$(0.3, 0.38, 0.32)$	$(0.2, 0.32, 0.48)$	$(0.1, 0.26, 0.64)$	$(0, 0.2, 0.8)$
12	$(0.6, 0.4, 0)$	$(0.48, 0.36, 0.16)$	$(0.36, 0.32, 0.32)$	$(0.24, 0.28, 0.48)$	$(0.12, 0.24, 0.64)$	$(0, 0.2, 0.8)$
13	$(0.4, 0.6, 0)$	$(0.36, 0.48, 0.16)$	$(0.32, 0.36, 0.32)$	$(0.28, 0.24, 0.48)$	$(0.24, 0.12, 0.64)$	$(0.2, 0, 0.8)$
14	$(0.5, 0.5, 0)$	$(0.44, 0.4, 0.16)$	$(0.38, 0.3, 0.32)$	$(0.32, 0.2, 0.48)$	$(0.26, 0.1, 0.64)$	$(0.2, 0, 0.8)$
15	$(0.6, 0.4, 0)$	$(0.52, 0.32, 0.16)$	$(0.44, 0.24, 0.32)$	$(0.36, 0.16, 0.48)$	$(0.28, 0.08, 0.64)$	$(0.2, 0, 0.8)$
16	$(0.7, 0.3, 0)$	$(0.6, 0.24, 0.16)$	$(0.5, 0.18, 0.32)$	$(0.4, 0.12, 0.48)$	$(0.3, 0.06, 0.64)$	$(0.2, 0, 0.8)$
17	$(0.8, 0.2, 0)$	$(0.68, 0.16, 0.16)$	$(0.56, 0.12, 0.32)$	$(0.44, 0.08, 0.48)$	$(0.32, 0.04, 0.64)$	$(0.2, 0, 0.8)$
18	$(0.2, 0.8, 0)$	$(0.16, 0.72, 0.12)$	$(0.12, 0.64, 0.24)$	$(0.08, 0.56, 0.36)$	$(0.04, 0.48, 0.48)$	$(0, 0.4, 0.6)$
19	$(0.3, 0.7, 0)$	$(0.24, 0.64, 0.12)$	$(0.18, 0.58, 0.24)$	$(0.12, 0.52, 0.36)$	$(0.06, 0.46, 0.48)$	$(0, 0.4, 0.6)$
20	$(0.4, 0.6, 0)$	$(0.32, 0.56, 0.12)$	$(0.24, 0.52, 0.24)$	$(0.16, 0.48, 0.36)$	$(0.08, 0.44, 0.48)$	$(0, 0.4, 0.6)$
21	$(0.5, 0.5, 0)$	$(0.4, 0.48, 0.12)$	$(0.3, 0.46, 0.24)$	$(0.2, 0.44, 0.36)$	$(0.1, 0.42, 0.48)$	$(0, 0.4, 0.6)$
22	$(0.5, 0.5, 0)$	$(0.48, 0.4, 0.12)$	$(0.46, 0.3, 0.24)$	$(0.44, 0.2, 0.36)$	$(0.42, 0.1, 0.48)$	$(0.4, 0, 0.6)$
23	$(0.6, 0.4, 0)$	$(0.56, 0.32, 0.12)$	$(0.52, 0.24, 0.24)$	$(0.48, 0.16, 0.36)$	$(0.44, 0.08, 0.48)$	$(0.4, 0, 0.6)$
24	$(0.7, 0.3, 0)$	$(0.64, 0.24, 0.12)$	$(0.58, 0.18, 0.24)$	$(0.52, 0.12, 0.36)$	$(0.46, 0.06, 0.48)$	$(0.4, 0, 0.6)$
25	$(0.8, 0.2, 0)$	$(0.72, 0.16, 0.12)$	$(0.64, 0.12, 0.24)$	$(0.56, 0.08, 0.36)$	$(0.48, 0.04, 0.48)$	$(0.4, 0, 0.6)$

Each lower envelope lottery listed above can be constructed as the convex combination $\delta R + (1 - \delta)Y$, where R represents the lower envelope lottery at the "Risky End" of the budget, and Y represents the lower envelope lottery at the "Uncertain End" of the budget.